Evolving Four-Bars for Optimal Synthesis

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Overview

A metric for Euclidean displacements

Evolution-based curve fitting

Evolving four-bars

Incorporating constraints

Conclusion and future research
The Euclidean space of affine displacements

Affine transformations

- $x \mapsto \alpha(x) = Ax + a$
- $\alpha \sim (A, a) \in \mathbb{R}^6$
- Euclidean motion group $SE(2) \subset \mathbb{R}^6$

Euclidean metric for affine transformations

- feature points $x_i \in \mathbb{R}^2$
- inner product $\langle \alpha, \beta \rangle := \sum \langle \alpha(x_i), \beta(x_i) \rangle$
- $\text{dist}^2(\alpha, \beta) = \langle \alpha - \beta, \alpha - \beta \rangle = \sum \|\alpha(x_i) - \beta(x_i)\|^2$
Properties of this metric

Advantages

• Euclidean
• left-invariant
• quite general (arbitrary mass distribution)
• easy to compute

Disadvantages

• need to specify feature points (mass distribution)
Motion design
Hofer, Pottmann, Ravani (2004)

Tolerancing in the Euclidean motion group
Schröcker, Wallner (2005)

Performance indices of robots
Nawratil (since 2006)

Penetration depth
Nawratil, Pottmann, Ravani (2007)
1. Shape parameter vector $s = s(t)$ (time dependent).
2. Closest points $p_j = p_j(t)$.
3. Ideally $q_j - p_j = \dot{p}_j$, that is,

$$\langle q_j - p_j, b_{jk} \rangle = \langle \dot{p}_j, b_{jk} \rangle = \cdots = - \sum_{i,l} \beta_{jki} \frac{\partial F_i}{\partial s_t} \dot{s}_l.$$

($b_{jk} = \sum_i \beta_{jki} \nabla F_i$ orthonormal basis of normal space in $p_j$).

$\implies$ Overconstrained linear system for the unknowns $\dot{s}_j$. 

\[ \text{Diagram:} \quad C, \quad p_0 \rightarrow q_0, \quad p_1 \rightarrow q_1, \quad p_2 \rightarrow q_2, \quad \ldots, \quad p_n \rightarrow q_n. \]
Curve $C$:  
- three equations describing $\mathrm{SE}(2) \subset \mathbb{R}^6$  
- two circle constraint equations  
  \[ \| \alpha(a_i, b_i) - (\xi_i, \eta_i) \|^2 = \rho_i^2 \]  

Target points $q_i$: prescribed poses on $\mathrm{SE}(2)$  
Closest points $p_i$: closest poses on the four-bar motion  
$C \subset \mathrm{SE}(2)$
McCarthyc’s eleven poses
McCarthy’s eleven poses
Circuit defect
Circuit defect
Circuit defect
Creeping in of circuit defects

Avoid double points on image curve during evolution!
Creeping in of circuit defects

Avoid double points on image curve $C$ during evolution!
Creeping in of circuit defects

Avoid double points on image curve $C$ during evolution!
Creeping in of circuit defects

Avoid double points on image curve during evolution!
Creeping in of circuit defects

Avoid double points on image curve during evolution!
Creeping in of circuit defects

Avoid double points on image curve C during evolution!
Folding four-bars

\[ P_1 = \frac{(a + b - c - d)^2}{(a + b + c + d)^2} > 0 \]
\[ P_2 = \frac{(a - b + c - d)^2}{(a + b + c + d)^2} > 0 \]
\[ P_3 = \frac{(a - b - c + d)^2}{(a + b + c + d)^2} > 0 \]

double points of \( C \) \( \iff \) folded positions of four-bar
Penalty functions

\[
P_1 = \frac{(a + b - c - d)^2}{(a + b + c + d)^2} \geq \delta_1 > 0
\]

\[
P_2 = \frac{(a - b + c - d)^2}{(a + b + c + d)^2} \geq \delta_2 > 0
\]

\[
P_3 = \frac{(a - b - c + d)^2}{(a + b + c + d)^2} \geq \delta_3 > 0
\]

(activator function) \cdot \left( \sum_l \frac{\partial P_i}{\partial s_l} \dot{s}_l - \omega_i(P_i) \right) = 0

Further linear equations for the unknowns \( \dot{s}_l \)!
Avoiding circuit defects
Avoiding circuit defects
Avoiding circuit defects
Conclusion and Future Research

Evolution based synthesis of four-bar mechanisms

- geometric significance
  - Euclidean design space with left-invariant metric
  - repeated closest point computation
  - geometrically motivated step-size
  - invariance with respect to choice of shape parameters
- well-established techniques (image processing, CAGD)
- constraints via penalty functions

Future research

- fast and robust closest point computation
- extend to other mechanism types
- incorporate more constraints (avoid collisions, preserve order, . . .)