Kinematic Mapping Based Assembly Mode Evaluation for Spherical Four-Bar Mechanisms

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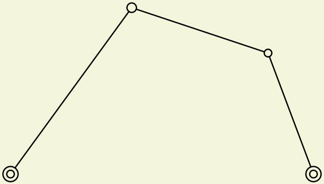
June 19, 2007

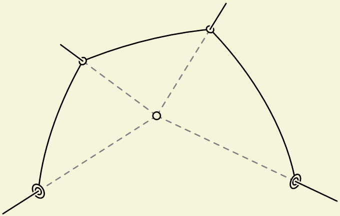
Outline

1. Assembly Mode Defective Four-Bars
2. A Principle of Transference for
   Planar and Spherical Circle Constraints
3. Assembly Mode Evaluation and Other Applications

Four-bar Mechanisms



planar four-bar

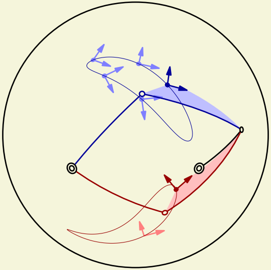


spherical four-bar

Synthesis problem

Find a four-bar that guides the coupler through a number of prescribed positions (precision points) $\mathbf{p}_0, \ldots, \mathbf{p}_n$.

Assembly mode defect



Problem statement

Find a simple, kinematic mapping based test to decide at an early stage in the design process whether two precision points lie in the same assembly mode or not.

Planar and Spherical Kinematic Mapping

$$\mathbf{A}(x_0, x_1, x_2, x_3) \mapsto [x_0, x_1, x_2, x_3]^T \in \mathbb{P}(\mathbb{R}^3)$$

Planar kinematic mapping

$$\mathbf{A} = \frac{1}{x_0^2 + x_3^2} \begin{bmatrix} x_0^2 + x_3^2 & 0 & 0 \\ 2(x_1x_3 + x_0x_2) & x_0^2 - x_3^2 & -2x_0x_3 \\ 2(x_2x_3 - x_0x_1) & 2x_0x_3 & x_0^2 - x_3^2 \end{bmatrix}$$

Spherical kinematic mapping

$$\mathbf{A} = \frac{1}{\Delta} \begin{bmatrix} x_0^2 + x_1^2 - x_2^2 - x_3^2 & 2(x_1x_2 - x_0x_3) & 2(x_1x_3 + x_0x_2) \\ 2(x_1x_2 + x_0x_3) & x_0^2 + x_2^2 - x_1^2 - x_3^2 & 2(x_2x_3 - x_0x_1) \\ 2(x_1x_3 - x_0x_2) & 2(x_0x_1 + x_2x_3) & x_0^2 + x_3^2 - x_1^2 - x_2^2 \end{bmatrix}$$

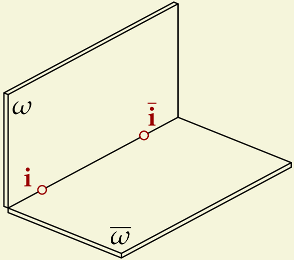
$$\text{where} \quad \Delta = x_0^2 + x_1^2 + x_2^2 + x_3^2$$

Circle Constraint Surfaces

The constraint surface of all planar displacements that transform a fixed point onto a fixed circle is a hyperboloid H.

Geometric Characterization

- H contains the points $\mathbf{i} = [0, 1, \mathrm{i}, 0]^T$, $\bar{\mathbf{i}} = [0, 1, -\mathrm{i}, 0]^T$
- H is tangent to the planes $\omega\colon x_0 + \mathrm{i}x_3 = 0$, $\overline{\omega}\colon x_0 - \mathrm{i}x_3 = 0$



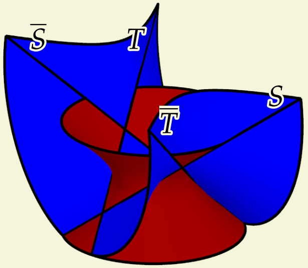
O. Bottema, B. Roth
Theoretical Kinematics
Dover Publications, 1990.
Chapter 11.

Kinematic Image of Spherical Four-bars

The constraint surface of all spherical displacements that transform a fixed point onto a fixed circle is a quadric Q.

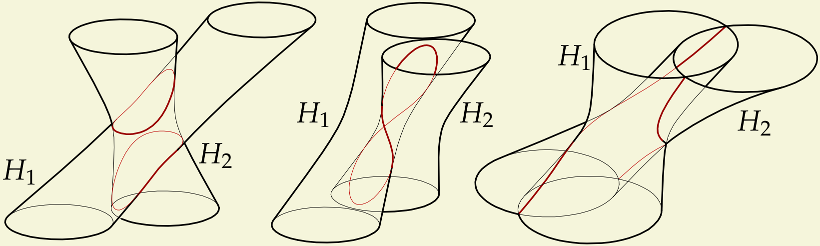
Geometric Characterization

- Q intersects the quadric $E\colon x_0^2 + x_1^2 + x_2^2 + x_3^2 = 0$ in two pairs S, $\overline{S}$, T, $\overline{T}$ of conjugate complex lines (Clifford quadric).



H. R. Müller
Sphärische Kinematik
VEB Deutscher Verlag der Wissenschaften, Berlin 1962
Chapter IV, §21.

Assembly Mode Evaluation of Four-bars



H.-P. Schröcker, M. Husty, and J. M. McCarthy
Kinematic Mapping Based Assembly Mode Evaluation of Planar Four-Bar Mechanisms
Journal of Mechanical Design (accepted for publication).

Assembly Mode Evaluation of Four-bars

Idea

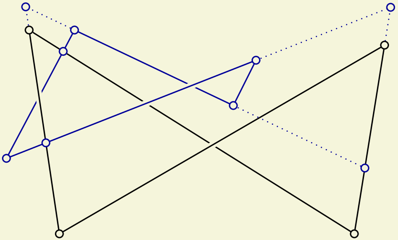
Find a real projective transformation

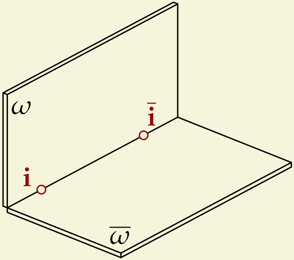
$$\alpha\colon \mathbb{P}(\mathbb{R}^3) \to \mathbb{P}(\mathbb{R}^3)$$

such that

$$\alpha(Q_i) = H_i, \quad i = 1, 2.$$

This transforms the spherical assembly mode problem to the planar case.





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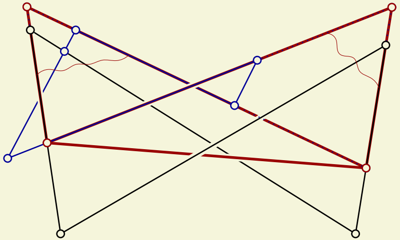

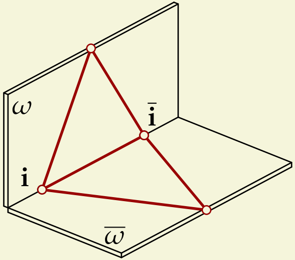

Assembly mode test

Given are a spherical four-bar (Clifford quadrics Q_1, Q_2) and two precision points $\mathbf{q}_1, \mathbf{q}_2 \in Q_1 \cap Q_2$.

1. Compute the real projective transformation α.
2. Transform the two Clifford quadrics Q_1, Q_2 to two circle constraint surfaces H_1, H_2 of planar kinematics.
3. Transform the two spherical precision points $\mathbf{q}_1, \mathbf{q}_2 \in Q_1 \cap Q_2$ to two planar precision points $\mathbf{h}_1, \mathbf{h}_2 \in H_1 \cap H_2$.
4. Perform the assembly mode test for H_1, H_2, $\mathbf{h}_1$ and $\mathbf{h}_2$.

Principle of Transference Between Planar and Spherical Circle Constraints

Further Applications

Principle of Transference Between
Planar and Spherical Circle Constraints

- assembly mode test
- workspace topology
- order of precision points
- motion parametrization
- ...

Families of Clifford Quadrics

- The lines S, $\overline{S}$, T, $\overline{T}$ are independent of the circle radius r.
- Different r yields Clifford quadrics $Q(r)$ with empty (real) intersection.
- Two families of Clifford quadrics $Q_1(r_1)$, $Q_2(r_2)$ can be mapped to two families of quadrics $H_1(r_1)$, $H_2(r_2)$ by a single real projective transformation α.

M. Husty
On the workspace of planar three-legged platforms.
Proceedings of ISRAM – World Congress of Automation
1790–1796, Montpellier, France, 1996.

A. Hofmeister, W. Sextro, O. Röschel
Error workspace analysis of planar mechanisms
Proceedings of EuCoMeS, Innsbruck 2006.

Summary

- The kinematic image of spherical four-bars is the intersection of two Clifford quadrics Q_1, Q_2.
- Two families of Clifford quadrics $Q_1(r_1)$, $Q_2(r_2)$ can be mapped to the kinematic images $H_1(r_1)$, $H_2(r_2)$ of planar four-bars by a real projective transformation.
- Transfer of results/techniques/computations from planar kinematics to spherical kinematics.