

Mixed area of n -gons

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We compute the symmetric matrix describing the mixed area form for n -gons.

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> restart: with(LinearAlgebra):
> n := 3:
> A := 0:
> for i from 2 to n do
  A := A + p[i-1,1] * p[i,2] - p[i-1,2] * p[i,1]:
od:
A := A + p[n,1] * p[1,2] - p[n,2] * p[1,1]:
A := 1/2 * A;

$$A := \frac{1}{2} p_{1,1} p_{2,2} - \frac{1}{2} p_{1,2} p_{2,1} + \frac{1}{2} p_{2,1} p_{3,2} - \frac{1}{2} p_{2,2} p_{3,1} + \frac{1}{2} p_{3,1} p_{1,2} - \frac{1}{2} p_{3,2} p_{1,1} \quad (1)$$

> M := Matrix(2*n, 2*n, shape=symmetric):
> for i from 1 to n do
  M[2*i-1,2*i-1] := coeff(A, p[i,1], 2):
  M[2*i,2*i] := coeff(A, p[i,2], 2):
  for j from i+1 to n do
    M[2*i-1,2*j-1] := 1/2 * coeff(coeff(A, p[i,1], 1), p[j,1],
1):
    M[2*i-1,2*j] := 1/2 * coeff(coeff(A, p[i,1], 1), p[j,2], 1):
    M[2*i,2*j-1] := 1/2 * coeff(coeff(A, p[i,2], 1), p[j,1], 1):
    M[2*i,2*j] := 1/2 * coeff(coeff(A, p[i,2], 1), p[j,2], 1):
  od:
od:
> M;
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$$\begin{bmatrix} 0 & 0 & 0 & \frac{1}{4} & 0 & -\frac{1}{4} \\ 0 & 0 & -\frac{1}{4} & 0 & \frac{1}{4} & 0 \\ 0 & -\frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & 0 & -\frac{1}{4} & 0 \\ 0 & \frac{1}{4} & 0 & -\frac{1}{4} & 0 & 0 \\ -\frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & 0 \end{bmatrix}$$

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