Fat polygons, ball coverings, and collision detection with toleranced motions

Matthias J. Weber
(joint work with Hans-Peter Schröcker)

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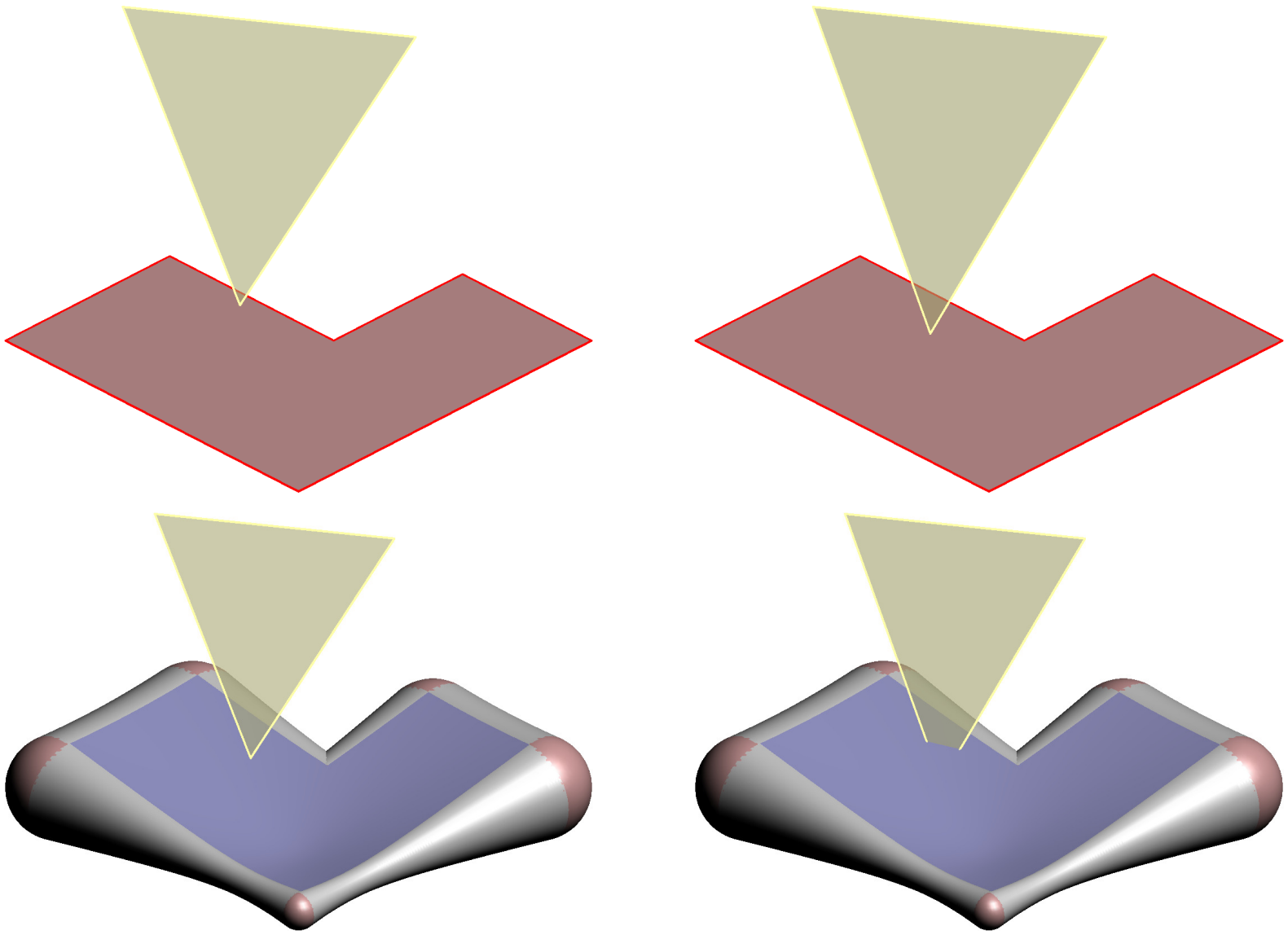
Toleranced motion



Preliminaries

affine mapping $\gamma\colon x \mapsto \gamma(x) = A \cdot x + a$, with $A \in \mathbb{R}^{3\times 3}$, $a \in \mathbb{R}^3$

identify the set of affine mappings with $\mathbb{R}^{12}$, that is, $\gamma \in \mathbb{R}^{12}$

H.-P. Schröcker, J. Wallner
Curvatures and tolerances in the Euclidean motion group.
Results Math. 47, 132–146, 2005.

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$$\langle \gamma, \beta \rangle = \int \langle \gamma(x), \beta(x) \rangle \, \mathrm{d}\mu(x)$$

μ ... positive Borel measure (mass distribution)

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identify the set of affine mappings with $\mathbb{R}^{12}$, that is, $\gamma \in \mathbb{R}^{12}$

$$\langle \gamma, \beta \rangle = \sum_{x \in F} \langle \gamma(x), \beta(x) \rangle$$

μ ... finite set of feature points F

mass distribution

μ ... finite set of unit mass feature points $F = \{x_1, \ldots, x_n\}$

associated:

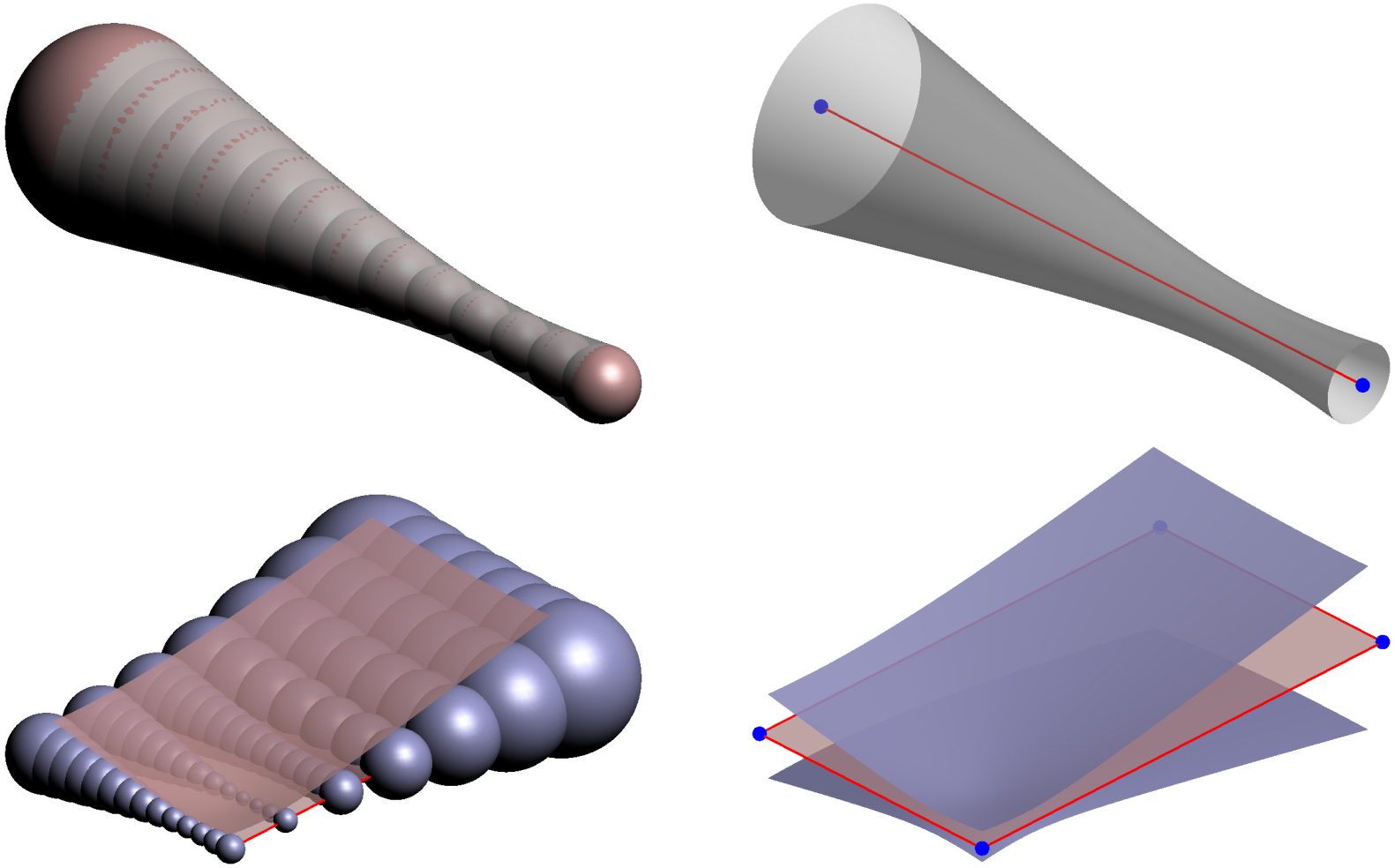
- barycenter $b = n^{-1} \sum x_i$,
- inertia matrix $J = \sum_{i=1}^{n} x_i \cdot x_i^T$,
- eigenvectors and eigenvalues of J,
- "normalized" coordinate system
  - origin $= b$,
  - axes ... given by eigenvectors and eigenvalues of J

Tolerance zones (envelopes of orbits)

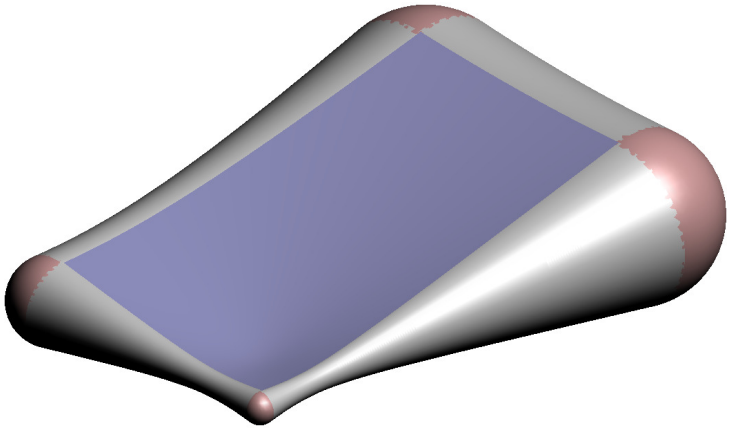
Orbits of a ball $\Gamma \in \mathbb{R}^{12}$ with radius R, center γ

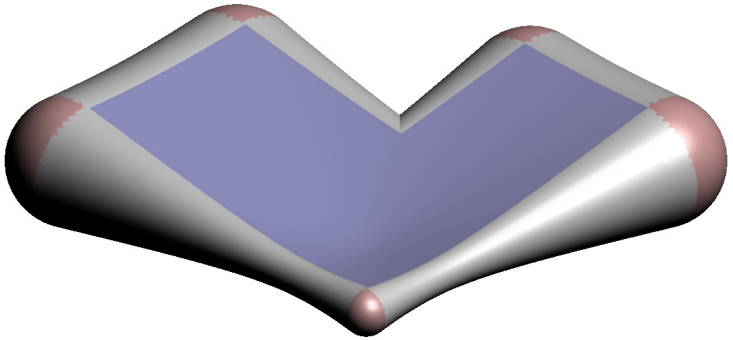
- Point x: Orbit is a ball with center $\gamma(x)$,
- Line l: Orbit is a hyperboloid of revolution with axis $\gamma(l)$,
- plane π: Orbit is a two sheeted hyperboloid with plane of symmetry $\gamma(\pi)$

Tolerance zones (envelopes of orbits)



Tolerance zone of a polygon



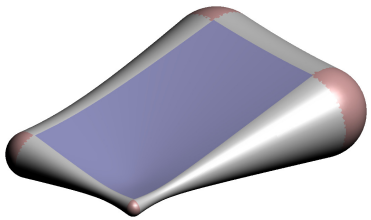


consists of:

- parts of spheres,
- parts of one sheeted hyperboloids of revolution,
- parts of a two sheeted hyperboloid.

Collision detection - exact

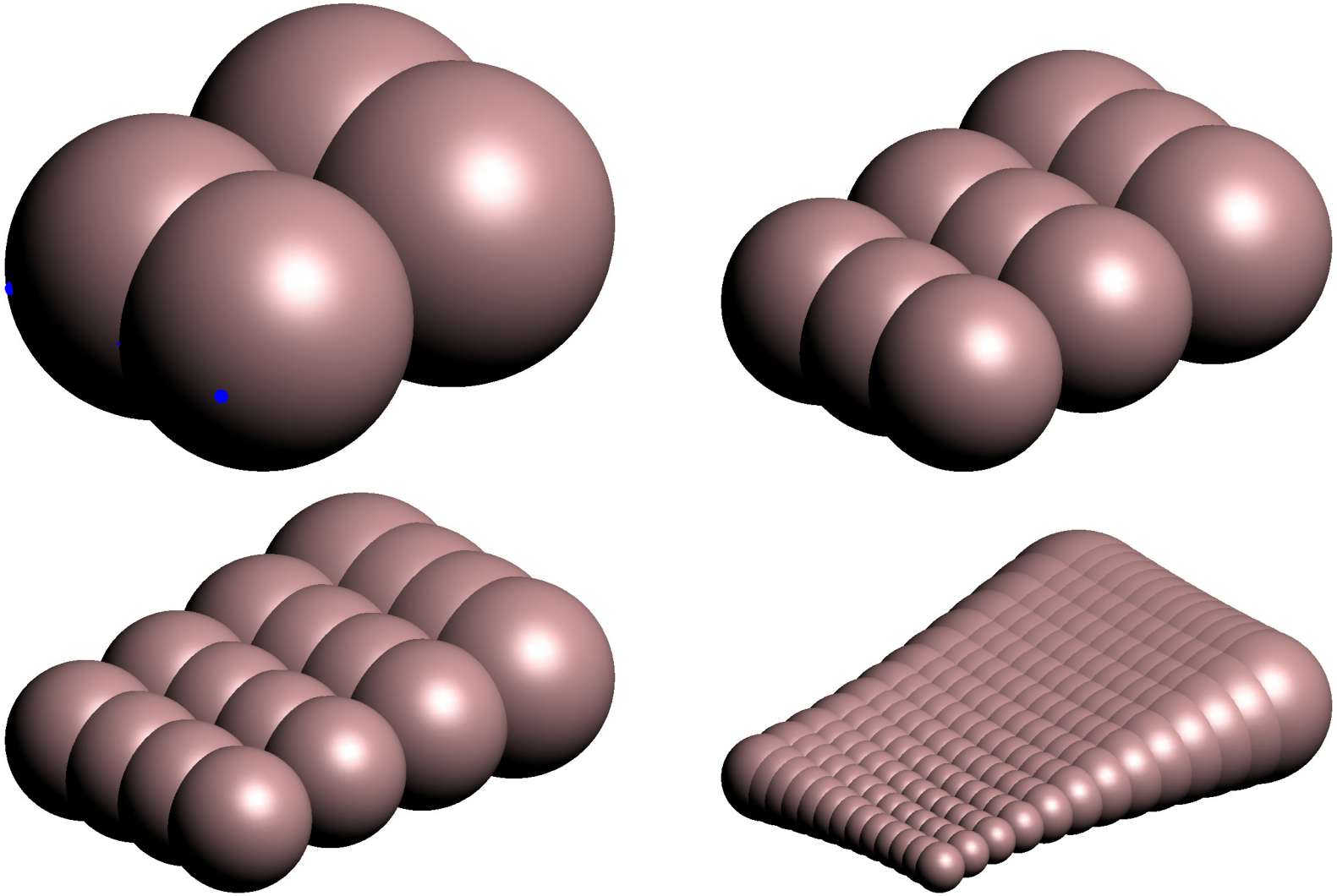
- sphere vs. polygon collision
- line segment intersects hyperboloid of revolution inside a pair of parallel planes
- point lies between the two sheets of a hyperboloid and the orthographic projection of the point lies inside the polygon



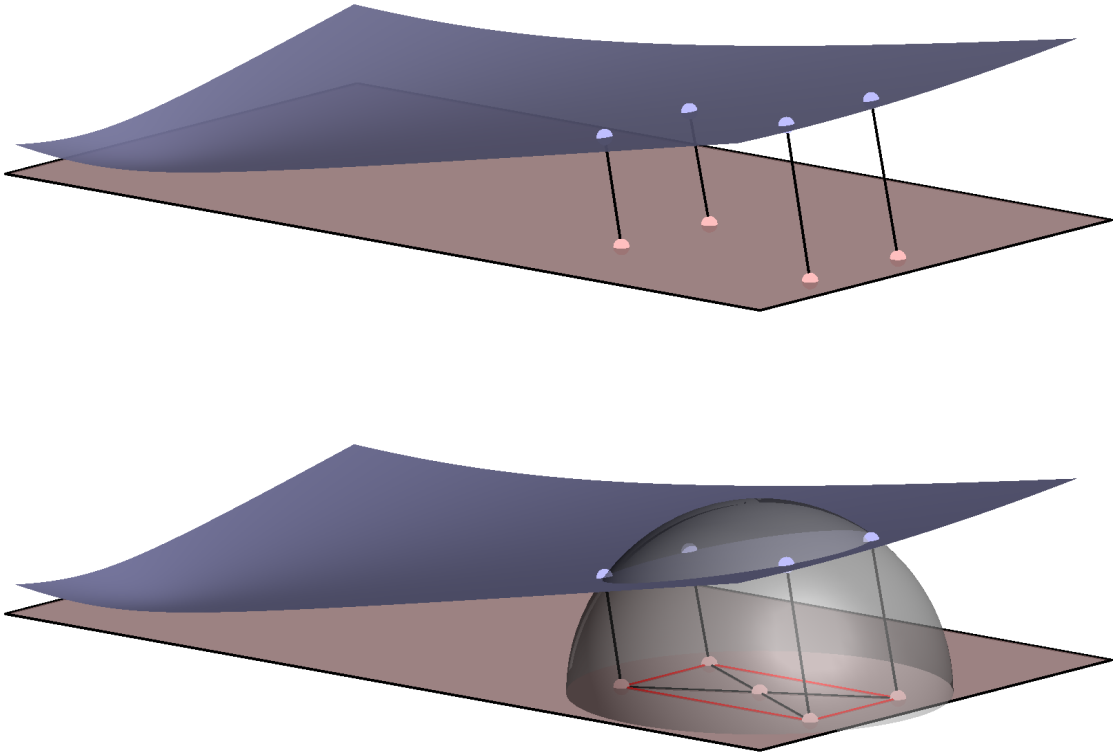
- positive: reduce R
- negative: collision can be excluded

Collision detection - sphere grid

grid based on bounding box of polygon



normal foot on the hyperboloid



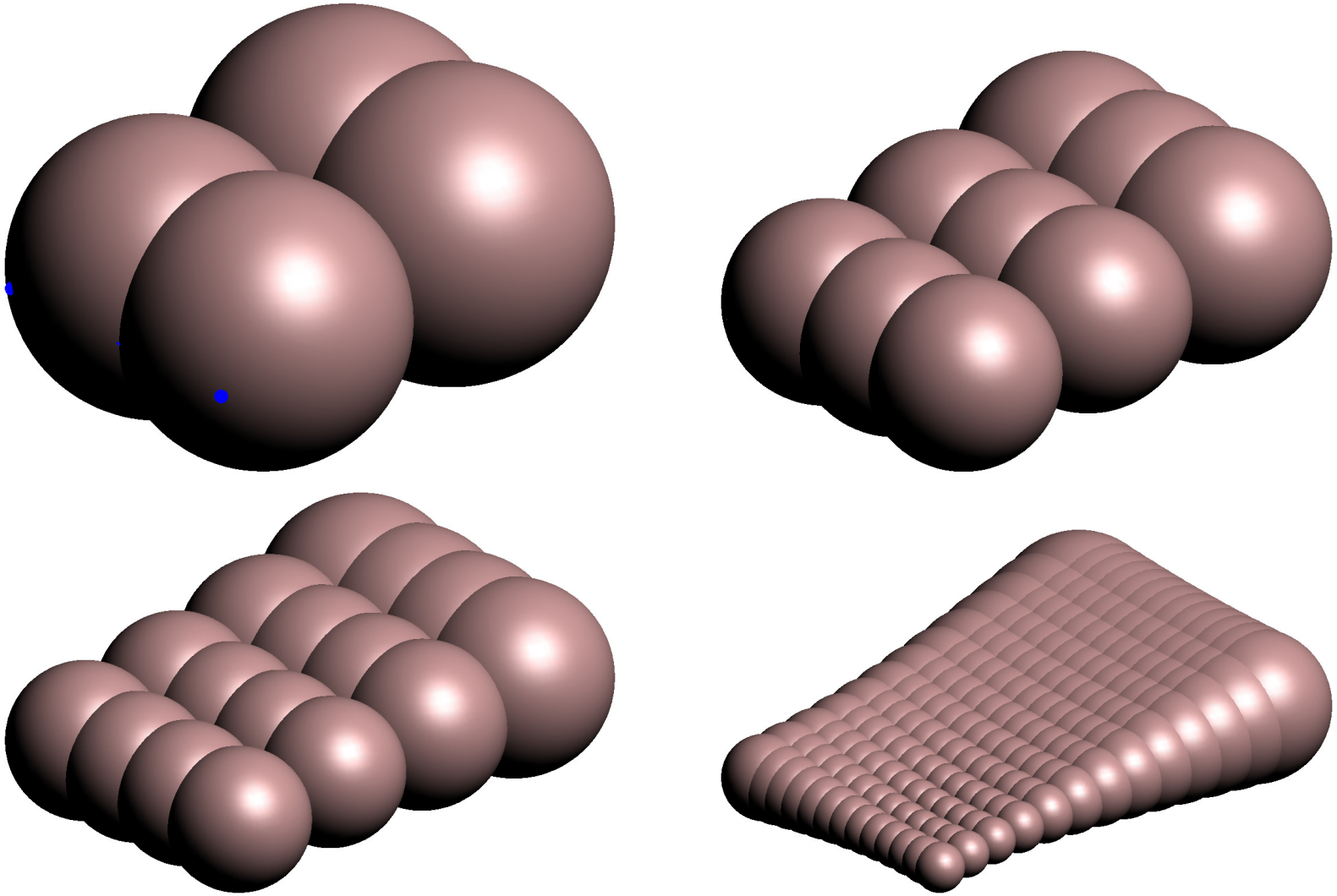
Theorem

The normal feet from the points $(x \pm d_x, y \pm d_y, 0)$ *on the hyperboloid*

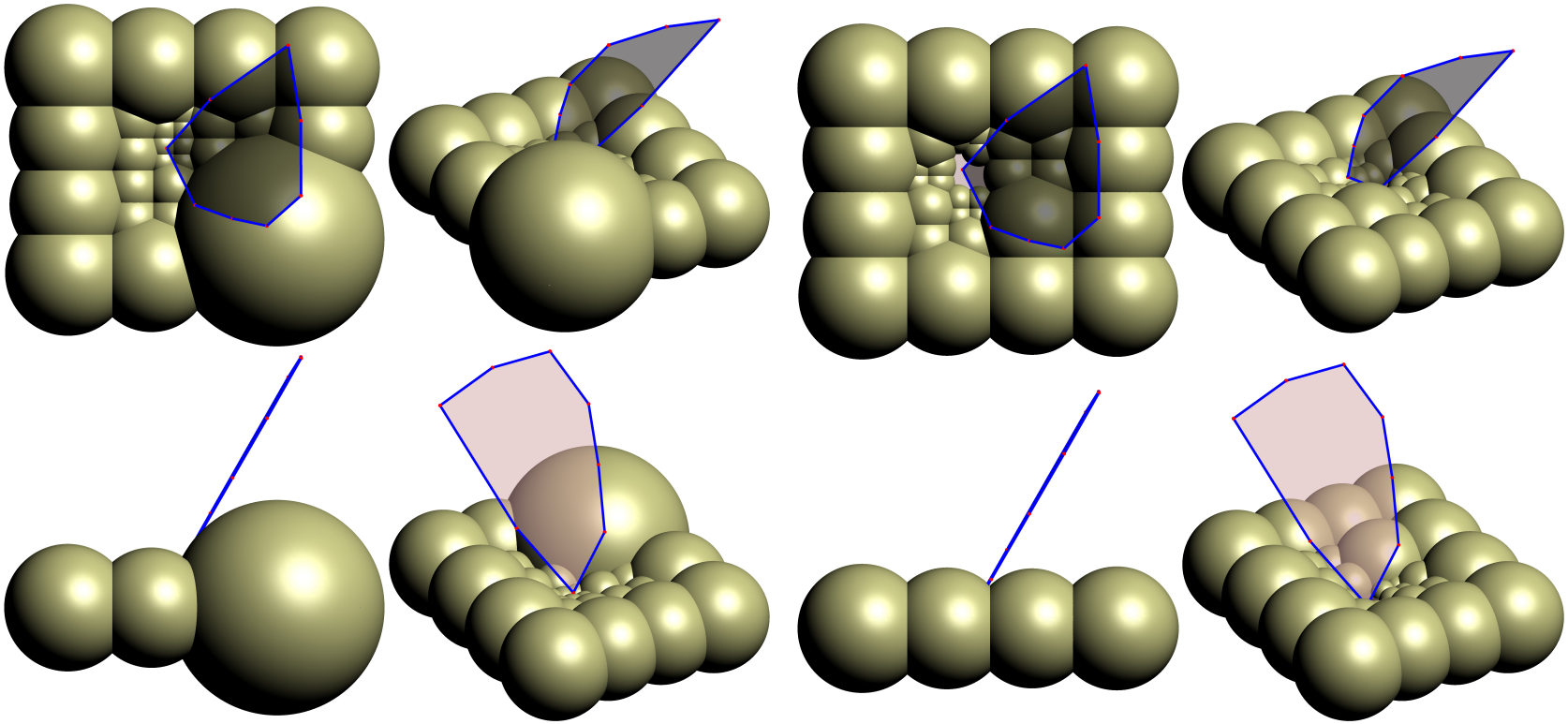
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = -1$$

lie on a ball with center $(x, y, 0)$.

sphere grid



Two examples



Algorithm

- Pre computing:
  metric data of the polygon P
  (barycenter, inertia matrix associated to μ)
- ball Γ gives R (start: $\gamma(t) \subset \Gamma$)
- compute envelope | sphere covering of the bounding box
- test for intersections
- / | subdivide necessary parts
- subdivide $\gamma(t)$, so that $\gamma(t) \subset \Gamma_1 \cap \Gamma_2 \rightarrow$ new R_1, R_2.