Fat polygons, ball coverings, and collision detection with tolerated motions

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Overview

Preliminaries

Details

Collision Detection - exact

Collision Detection - sphere grid

Algorithm / work in progress
Toleranced motion
Preliminaries

affine mapping $\gamma: x \mapsto \gamma(x) = A \cdot x + a$, with $A \in \mathbb{R}^{3 \times 3}$, $a \in \mathbb{R}^{3}$

identify the set of affine mappings with $\mathbb{R}^{12}$, that is, $\gamma \in \mathbb{R}^{12}$

H.-P. Schröcker, J. Wallner
Curvatures and tolerances in the Euclidean motion group.
Preliminaries

affine mapping $\gamma: x \mapsto \gamma(x) = A \cdot x + a$, with $A \in \mathbb{R}^{3 \times 3}$, $a \in \mathbb{R}^3$

identify the set of affine mappings with $\mathbb{R}^{12}$, that is, $\gamma \in \mathbb{R}^{12}$

$$\langle \gamma, \beta \rangle = \int \langle \gamma(x), \beta(x) \rangle \, d\mu(x)$$

$\mu$ ... positive Borel measure (mass distribution)
Preliminaries

affine mapping $\gamma: x \mapsto \gamma(x) = A \cdot x + a$, with $A \in \mathbb{R}^{3 \times 3}, a \in \mathbb{R}^3$

identify the set of affine mappings with $\mathbb{R}^{12}$, that is, $\gamma \in \mathbb{R}^{12}$

$$\langle \gamma, \beta \rangle = \sum_{x \in F} \langle \gamma(x), \beta(x) \rangle$$

$\mu \ldots$ finite set of feature points $F$
mass distribution

\[ \mu \ldots \text{finite set of unit mass feature points} \quad F = \{ x_1, \ldots, x_n \} \]

associated:
- barycenter \( \mathbf{b} = n^{-1} \sum x_i \),
- inertia matrix \( \mathbf{J} = \sum_{i=1}^{n} x_i \cdot x_i^T \),
- eigenvectors and eigenvalues of \( \mathbf{J} \),
- “normalized” coordinate system
  - origin = \( \mathbf{b} \),
  - axes \ldots \text{given by eigenvectors and eigenvalues of } \mathbf{J}
Tolerance zones (envelopes of orbits)

Orbits of a ball $\Gamma \in \mathbb{R}^{12}$ with radius $R$, center $\gamma$

- **Point $x$:** Orbit is a ball with center $\gamma(x)$,
- **Line $l$:** Orbit is a hyperboloid of revolution with axis $\gamma(l)$,
- **plane $\pi$:** Orbit is a two sheeted hyperboloid with plane of symmetry $\gamma(\pi)$
Tolerance zones (envelopes of orbits)
Tolerance zone of a polygon consists of:

- parts of spheres,
- parts of one sheeted hyperboloids of revolution,
- parts of a two sheeted hyperboloid.
Collision detection - exact

- sphere vs. polygon collision
- line segment intersects hyperboloid of revolution inside a pair of parallel planes
- point lies between the two sheets of a hyperboloid and the orthographic projection of the point lies inside the polygon

- positive: reduce $R$
- negative: collision can be excluded
Collision detection - sphere grid

grid based on bounding box of polygon
normal foot on the hyperboloid

**Theorem**

*The normal feet from the points* $(x \pm d_x, y \pm d_y, 0)$ *on the hyperboloid*

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = -1$$

*lie on a ball with center* $(x, y, 0)$.
sphere grid
Two examples
Pre computing:
- metric data of the polygon $P$
  (barycenter, inertia matrix associated to $\mu$)
- ball $\Gamma$ gives $R$ (start: $\gamma(t) \subset \Gamma$)
- compute envelope | sphere covering of the bounding box
- test for intersections
- / | subdivide necessary parts
- subdivide $\gamma(t)$, so that $\gamma(t) \subset \Gamma_1 \cap \Gamma_2 \rightarrow \text{new } R_1, R_2$. 