Evolving Four-Bars for Optimal Synthesis

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2nd European Conference on Mechanism Science
Cassino, September 2008

Overview

A metric for Euclidean displacements

Evolution-based curve fitting

Evolving four-bars

Incorporating constraints

Conclusion and future research

The Euclidean space of affine displacements

Affine transformations

- $x \mapsto \alpha(x) = Ax + a$
- $\alpha \rightsquigarrow (A, a) \in \mathbb{R}^6$
- Euclidean motion group $\mathrm{SE}(2) \subset \mathbb{R}^6$

Euclidean metric for affine transformations

- feature points $x_i \in \mathbb{R}^2$
- inner product $\langle \alpha, \beta \rangle := \sum \langle \alpha(x_i), \beta(x_i) \rangle$
- $\mathrm{dist}^2(\alpha, \beta) = \langle \alpha - \beta, \alpha - \beta \rangle = \sum \|\alpha(x_i) - \beta(x_i)\|^2$

Properties of this metric

Advantages

- Euclidean
- left-invariant
- quite general (arbitrary mass distribution)
- easy to compute

Disadvantages

- need to specify feature points (mass distribution)

Applications in Computational Kinematics

Motion design

Hofer, Pottmann, Ravani (2004)

Tolerancing in the Euclidean motion group

Schröcker, Wallner (2005)

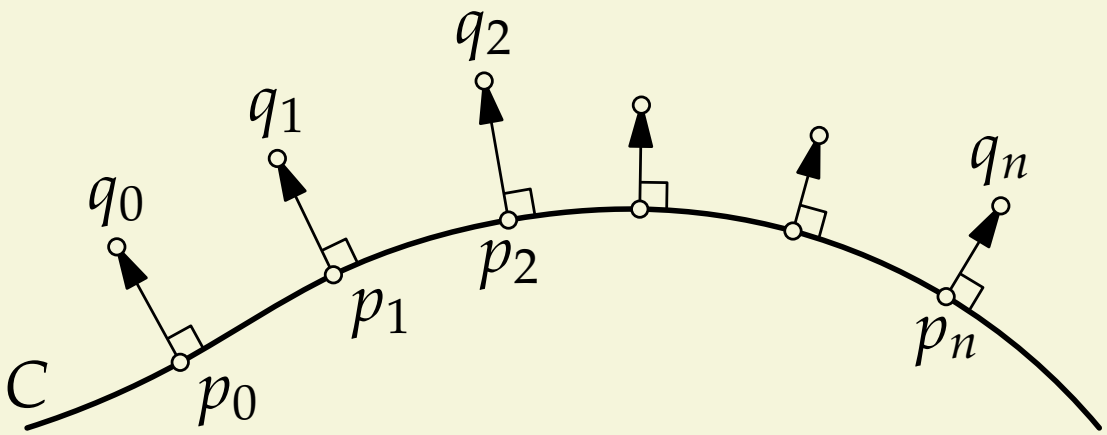
Performance indices of robots

Nawratil (since 2006)

Penetration depth

Nawratil, Pottmann, Ravani (2007)

Curve and surface evolution (M. Aigner, B. Jüttler)



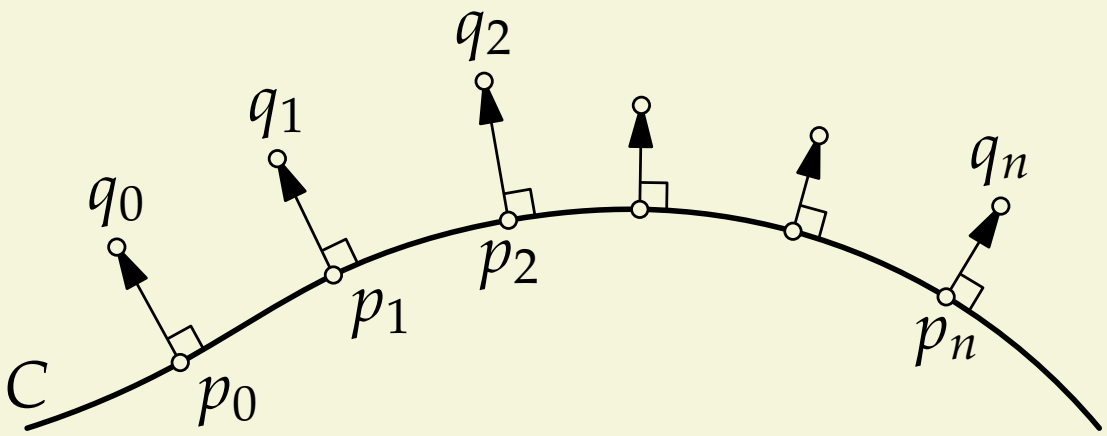
1. Shape parameter vector $s = s(t)$ (time dependent).
2. Closest points $p_j = p_j(t)$.
3. Ideally $q_j - p_j = \dot{p}_j$, that is,

$$\langle q_j - p_j, b_{jk} \rangle = \langle \dot{p}_j, b_{jk} \rangle = \cdots = -\sum_{i,l} \beta_{jki} \frac{\partial F_i}{\partial s_t} \dot{s}_l.$$

($b_{jk} = \sum_i \beta_{jki} \nabla F_i$ orthonormal basis of normal space in p_j).

$\Longrightarrow$ Overconstrained linear system for the unknowns $\dot{s}_j$.

Four-Bar evolution

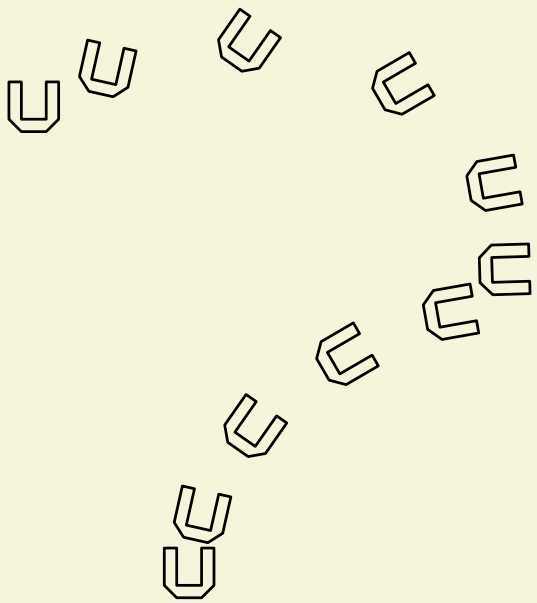


Curve C:
- three equations describing $\mathrm{SE}(2) \subset \mathbb{R}^6$
- two circle constraint equations $\|\alpha(a_i, b_i) - (\xi_i, \eta_i)\|^2 = \rho_i^2$

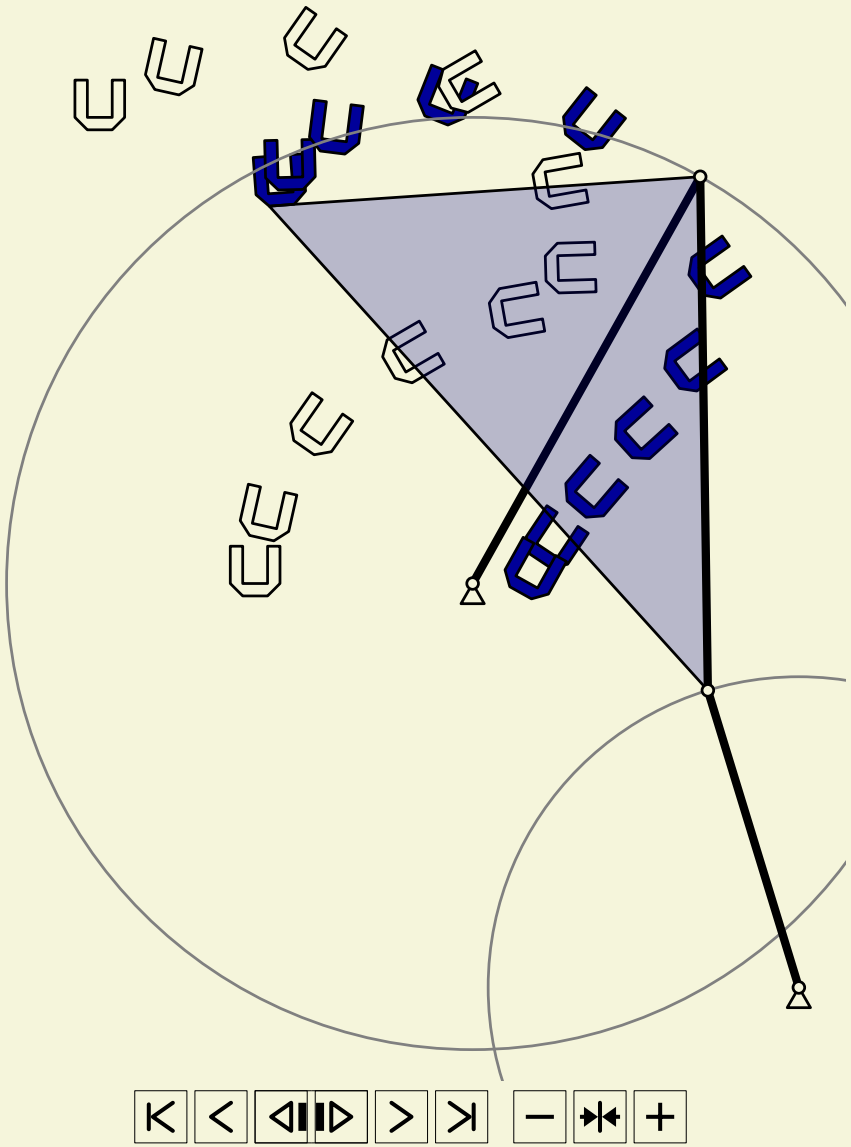
Target points q_i: prescribed poses on $\mathrm{SE}(2)$

Closest points p_i: closest poses on the four-bar motion $C \subset \mathrm{SE}(2)$

McCarthy's eleven poses



McCarthy's eleven poses



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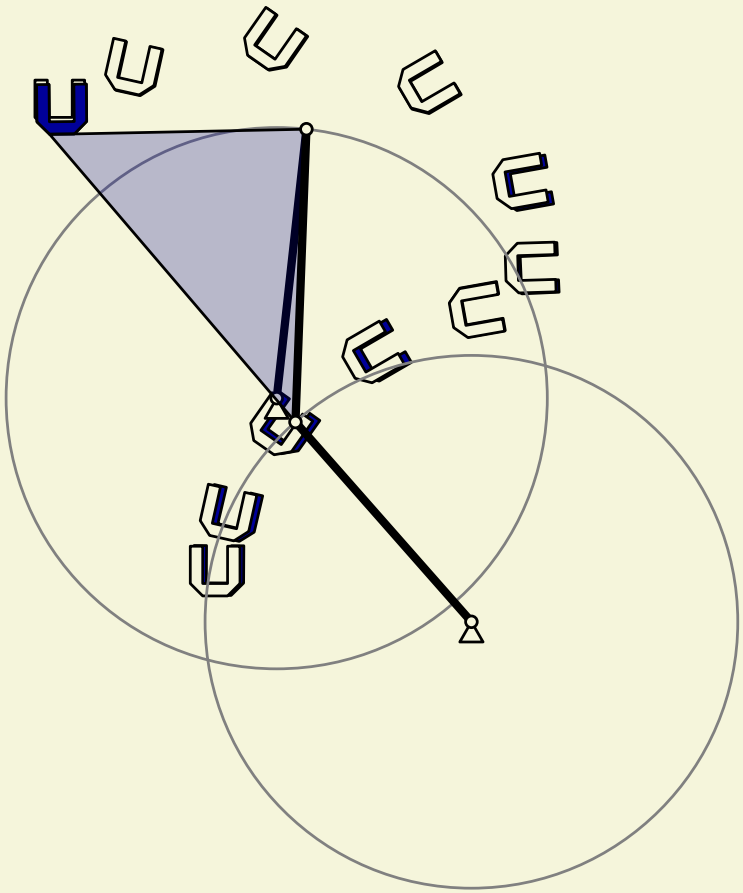
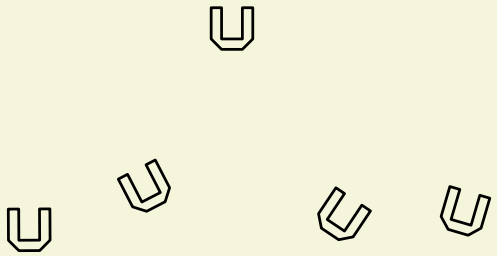


Circuit defect



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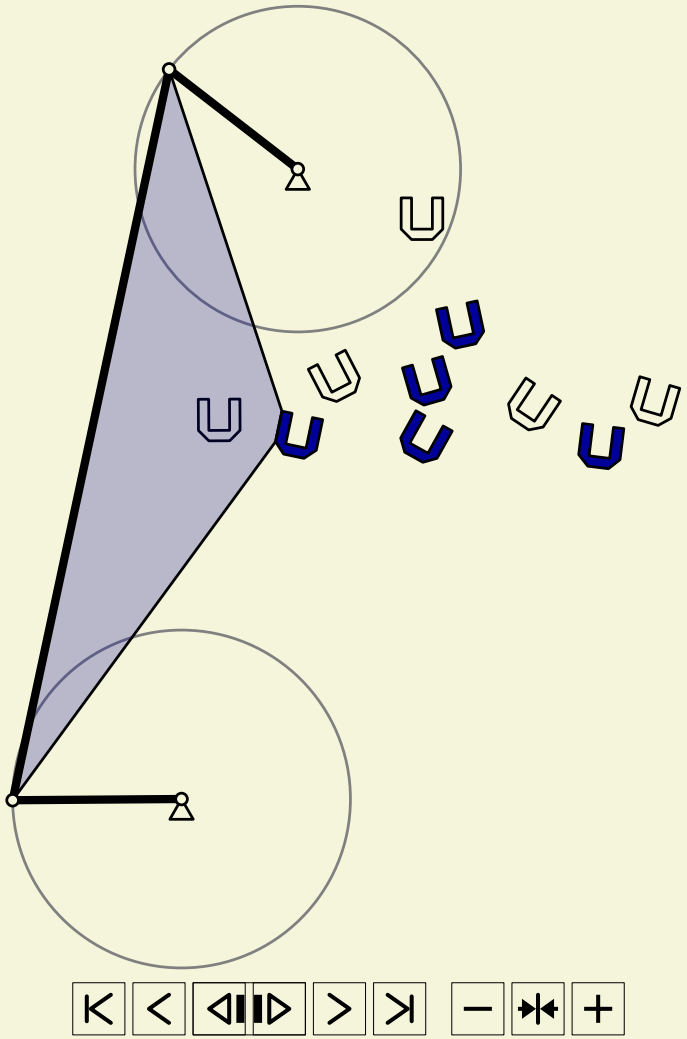
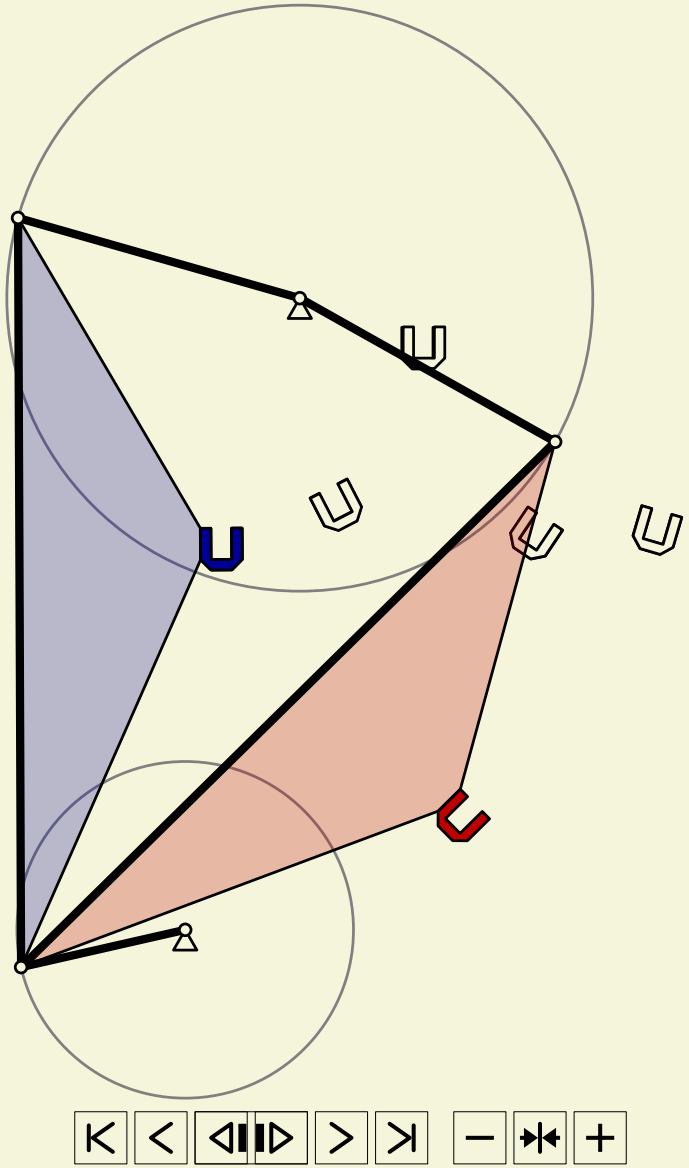
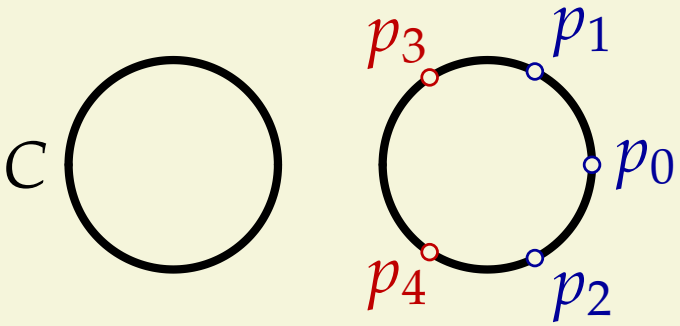


Circuit defect



Creeping in of circuit defects



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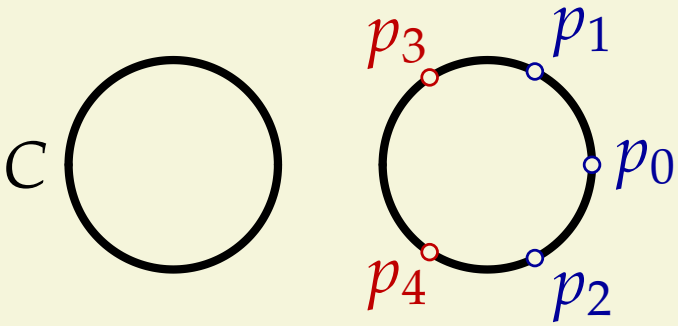

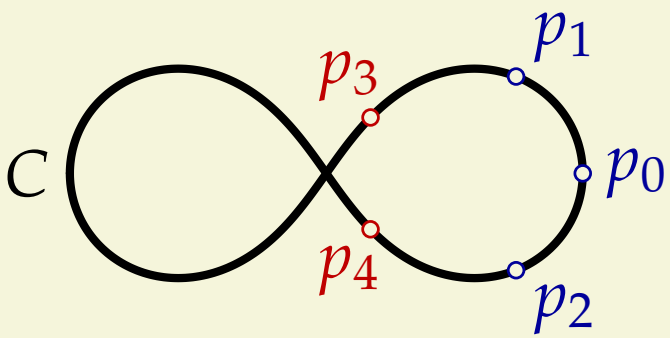

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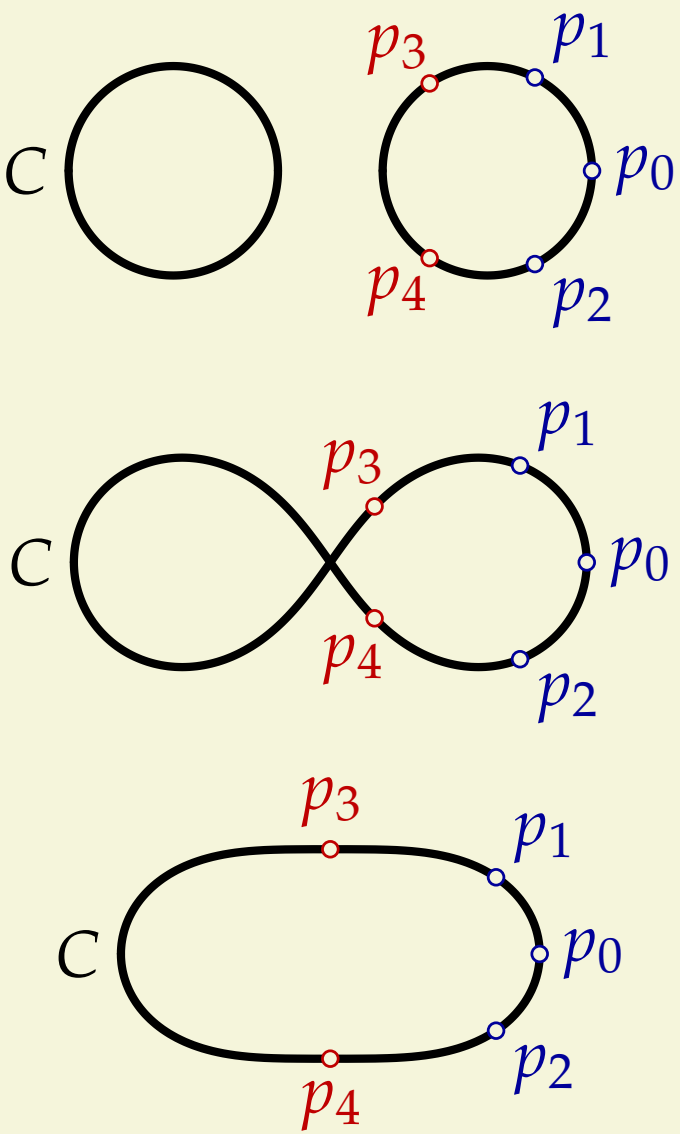

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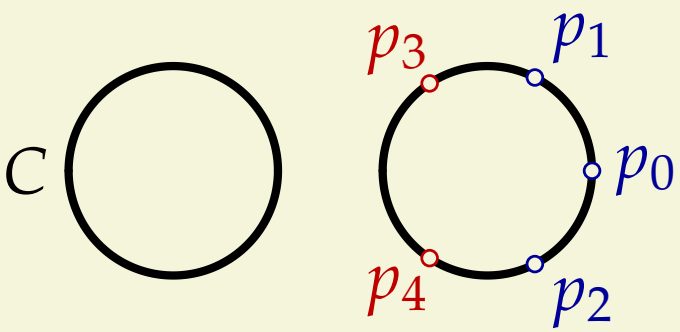

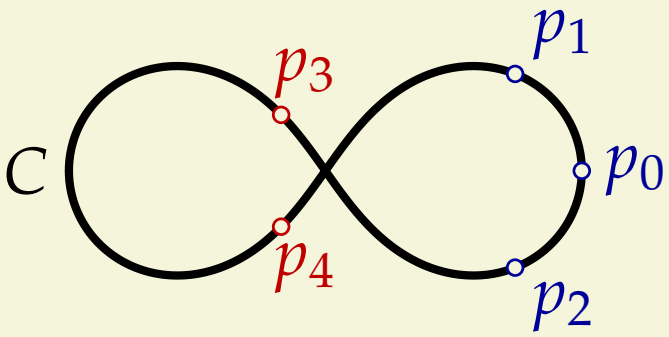

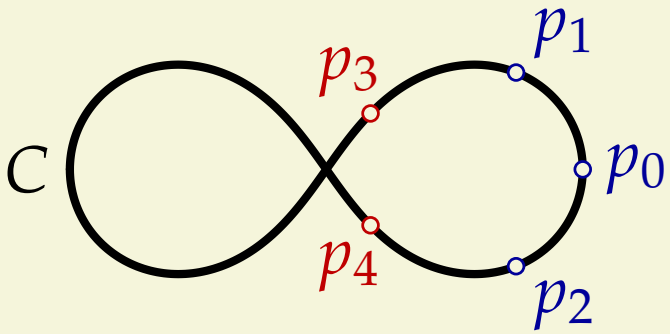

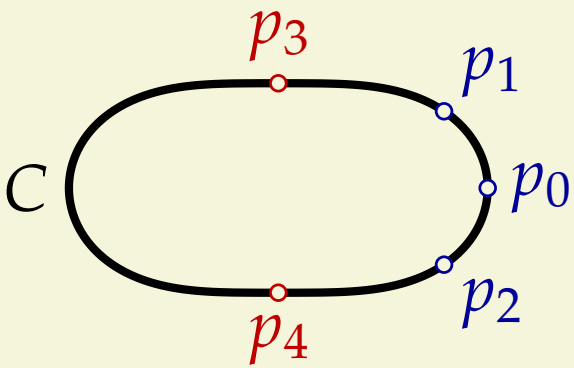

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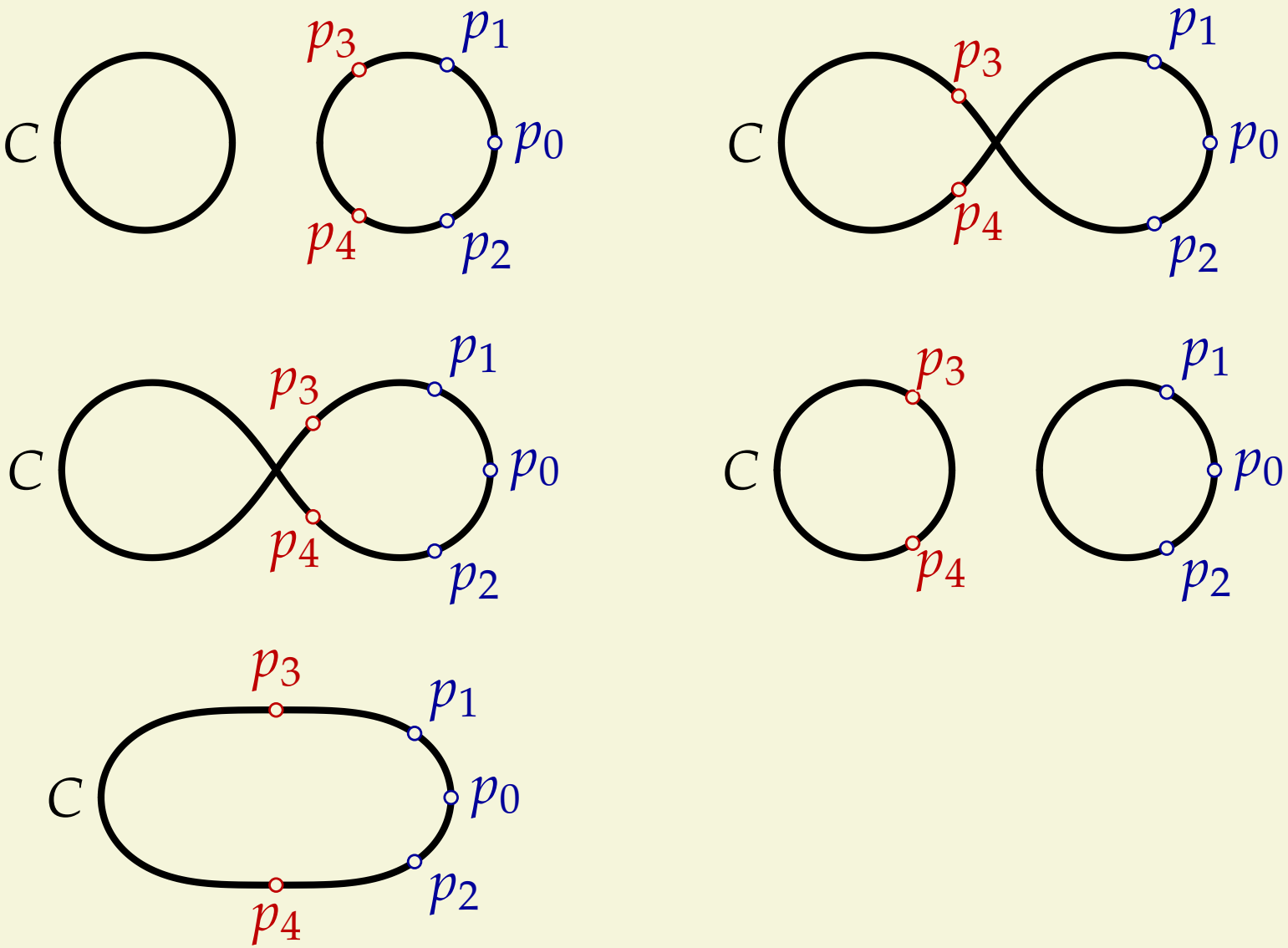

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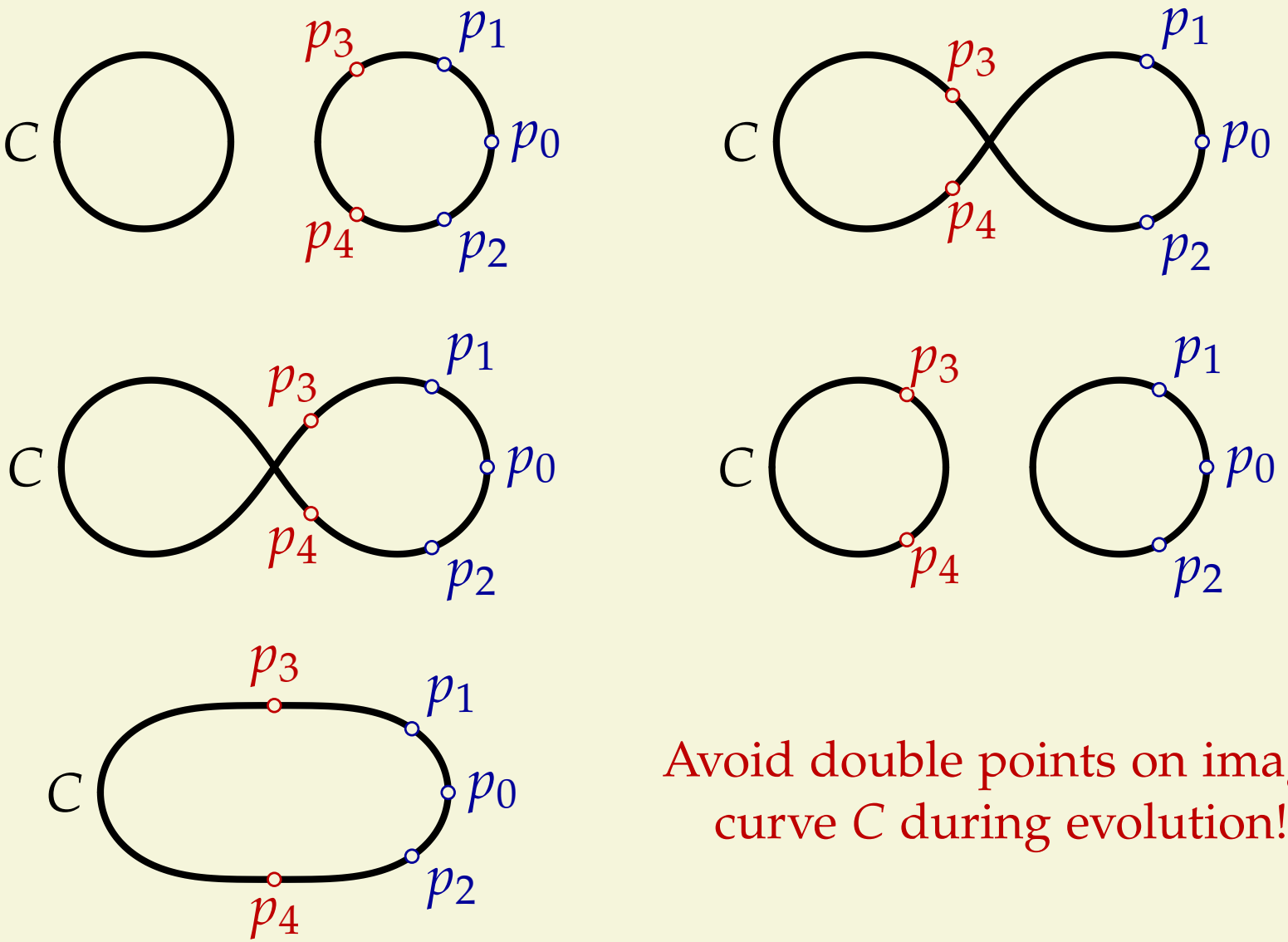
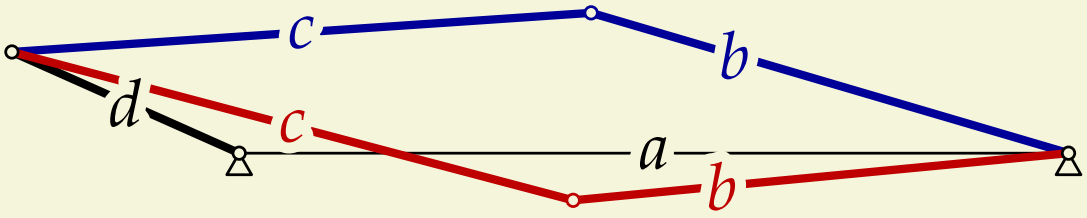


Avoid double points on image curve C during evolution!

Folding four-bars



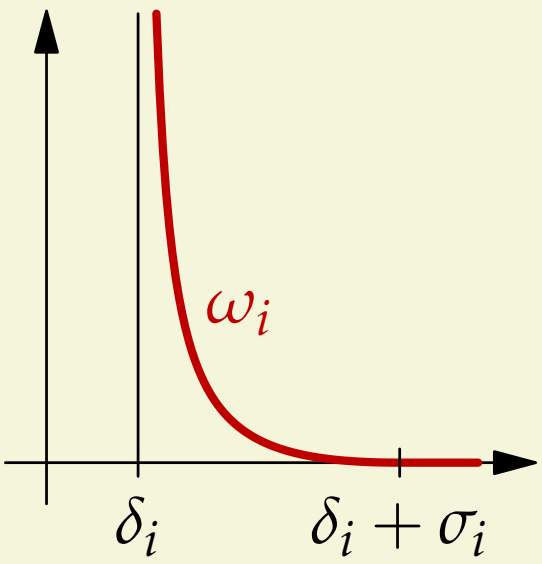
double points of $C \iff$ folded positions of four-bar

$$P_1 = \frac{(a+b-c-d)^2}{(a+b+c+d)^2} > 0$$

$$P_2 = \frac{(a-b+c-d)^2}{(a+b+c+d)^2} > 0$$

$$P_3 = \frac{(a-b-c+d)^2}{(a+b+c+d)^2} > 0$$

Penalty functions



$$P_1 = \frac{(a+b-c-d)^2}{(a+b+c+d)^2} \geqslant \delta_1 > 0$$

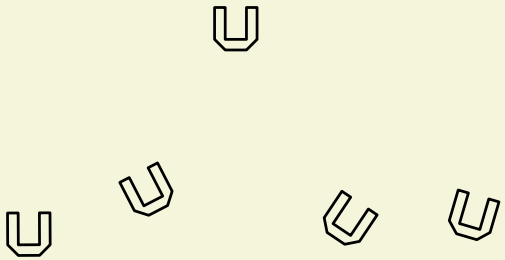
$$P_2 = \frac{(a-b+c-d)^2}{(a+b+c+d)^2} \geqslant \delta_2 > 0$$

$$P_3 = \frac{(a-b-c+d)^2}{(a+b+c+d)^2} \geqslant \delta_3 > 0$$

$$\text{(activator function)} \cdot \left(\sum_l \frac{\partial P_i}{\partial s_l} \dot{s}_l - \omega_i(P_i) \right) = 0$$

Further linear equations for the unknowns $\dot{s}_l$!

Avoiding circuit defects



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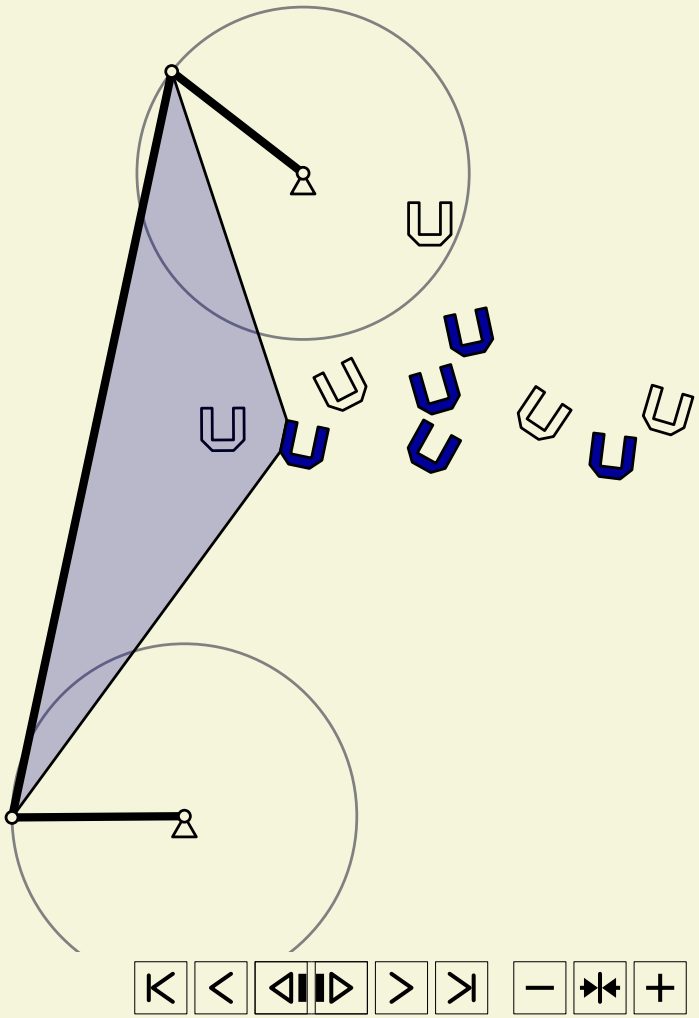

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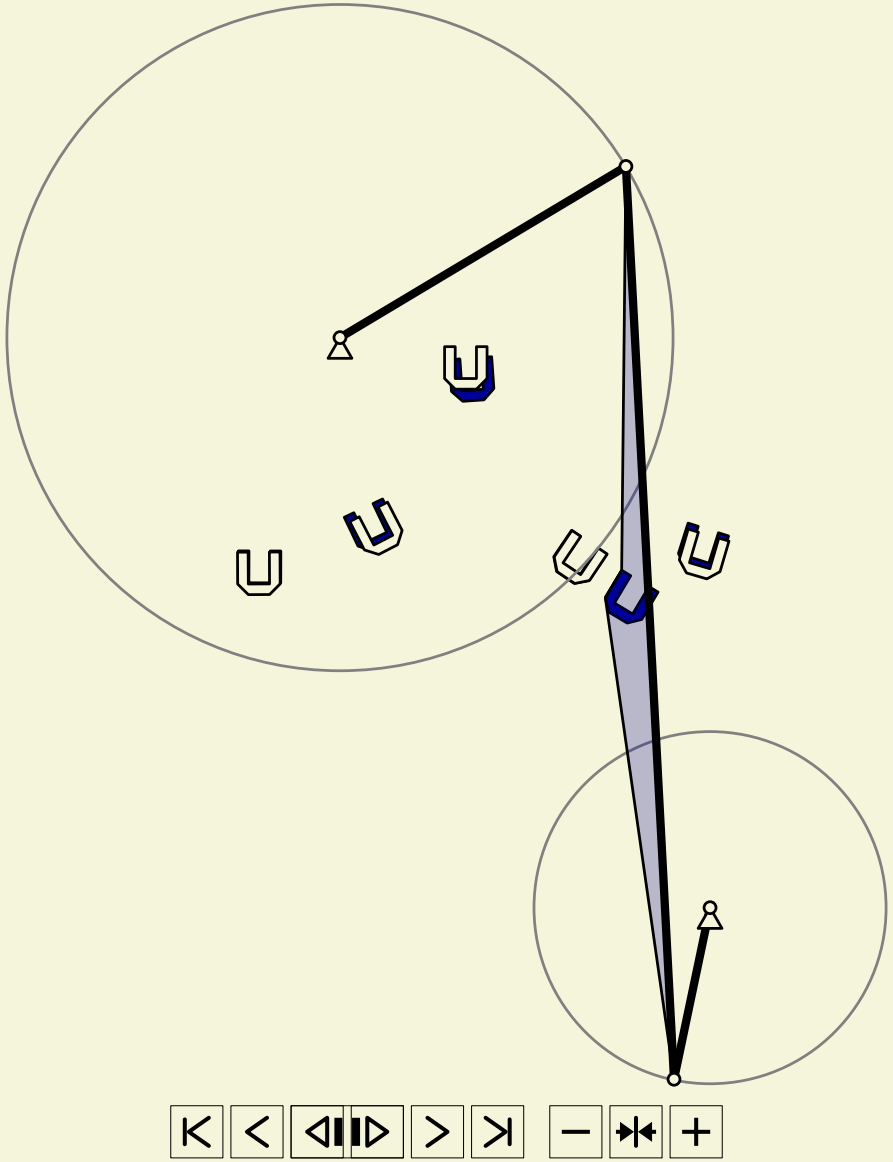

Conclusion and Future Research

Evolution based synthesis of four-bar mechanisms

- geometric significance
  - Euclidean design space with left-invariant metric
  - repeated closest point computation
  - geometrically motivated step-size
  - invariance with respect to choice of shape parameters
- well-established techniques (image processing, CAGD)
- constraints via penalty functions

Future research

- fast and robust closest point computation
- extend to other mechanism types
- incorporate more constraints (avoid collisions, preserve order, ...)