WALTER WUNDERLICH (1910–1998)

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Abstract. Walter Wunderlich was one of the most influential Austrian kinematicians in the 20th century. He wrote more than 200 scientific papers in the fields of mathematics, geometry and kinematics. Because of his influence, kinematic geometry is still an important subject in the curricula of geometry teachers' education in Austria.

Biographical Notes

Walter Wunderlich was born in Vienna on March 6th, 1910.[1] His father was an engineer. His ancestors came from different parts of the former Austrian empire. He had relatives in today's Slovenia, Bohemia and Hungary. These relatives were very important in his youth. Many of his holidays were spent in Hungary or in Bohemia, where he learned the Hungarian and Czech languages, both of which he spoke fluently.

After graduation from the "Realschule" (a natural science oriented high school) he studied civil engineering, but after finishing the undergraduate courses he changed to mathematics and descriptive geometry. He wanted to become a high school teacher in these subjects. At the University of Vienna he had excellent and famous professors as teachers. In mathematics: P. Furtwängler (founder of the Vienna school of number theory), H. Hahn (one of the founders of functional analysis, "theorem of Hahn–Banach"), K. Mayrhofer and W. Wirtinger (functional analysis, Wirtinger was the third winner of the Cayley prize after Cantor and Poincaré) and in descriptive geometry L. Eckhart, J. Krames, E. Kruppa, Th. Schmid and L. Schrutka. His master's

[1] Biographical notes are taken from Stachel (1999).

Fig. 1. Walter Wunderlich in March 1990. (Photo taken from Stachel (1999) with permission from the author)

thesis (Diplomarbeit) under supervision of E. Kruppa on "Nichteuklidische Schraubungen" (Non-Euclidean Screw Motions) was finished in 1933 and already one year later he submitted his PhD. thesis (Doktorarbeit) having the title "Über eine affine Verallgemeinerungen der Lyon'schen Grenzschraubungen" (An affine generalization of Lyon's limit screw-motions).

Surrounded by many problems, mainly due to the political situation in Austria, he started a scientific career, got a part-time position as a tutor and in 1934 became assistant to L. Eckhart. In 1939 he submitted a habilitation thesis with the title "Darstellende Geometrie nichteuklidischer Schraubflächen" (Descriptive geometry of non-Euclidean screw surfaces). Immediately after the habilitation exam he was conscripted to the army. This fact is mentioned because it was during the war that he wrote his first scientific papers. Three of his papers even bear the affiliation "in the battlefield". In 1942 he was released from military service and went as a private researcher to a military research institution. He worked in the unit "physics of blasting" and wrote a book titled *Introduction to Under-Water Blasting*, which was finished after the World War under American supervision.[2] In 1943 he became university docent in Berlin, but because of problems due to the war he never started teaching. After the war he and his wife were in British internment

[2] He himself never mentions this book in the list of publications.

camps, where also their first son was born. In 1946, after coming back to Vienna, he immediately was re-employed at TU Vienna because he never was politically active. He became associate professor at the Technical University in Vienna and in 1951 he was appointed full professor. From that time he always stayed in Vienna, although many universities offered him challenging positions: Karlsruhe, Aachen and Munich. He was Dean of Natural Sciences faculty and Rector Magnificus of TU Vienna. He retired in 1980, but wrote more than 40 high standard scientific papers as a professor emeritus. Ten years before his death he suffered a retina ablation and died almost blind in 1998.

Walter Wunderlich was a member of the Austrian academy of science and for more than 25 years honorary editor of the IFToMM journal Mechanism and Machine Theory.

List of Main Works

Books

Wunderlich, W., *Ebene Kinematik*.
Wunderlich, W., *Darstellende Geometrie I*, 1966.
Wunderlich, W., *Darstellende Geometrie II*, 1967.

Contributions to Non-Euclidean Kinematics

Wunderlich, W., Über eine affine Verallgemeinerung der Grenzschraubung, 1935.
Wunderlich, W., Darstellende Geometrie nichteuklidischer Schraubflächen, 1936.

Contributions to CAM Theory

Wunderlich, W., Contributions to the geometry of cam mechanisms with oscillating followers, 1971.
Wunderlich, W., Kurven mit isoptischem Kreis, 1971.
Wunderlich, W., Kurven mit isoptischer Ellipse, 1971.
Wunderlich, W., Single-disk cam mechanisms with oscillating double roller follower, 1984.

Contributions to Gearing Theory

Wunderlich, W. and Zenow, P., Contribution to the geometry of elliptic gears, 1975.
Wunderlich, W., Zur Triebstockverzahnung, 1943.
Wunderlich, W., Über abwickelbare Zahnflanken und eine neue Kegelradverzahnung, 1948.

Contributions to Singular Mechanisms

Wunderlich, W., Starre, kippende, wackelige und bewegliche Achtflache, 1965.
Wunderlich, W., Starre, kippende, wackelige und bewegliche Gelenkvierecke im Raum, 1971.
Wunderlich, W., Fokalkurvenpaare in orthogonalen Ebenen und bewegliche Stabwerke, 1976.
Wunderlich, W., Über die gefährlichen Örter bei zwei Achtpunktproblemen und einem Fünfpunktproblem, 1977.
Wunderlich, W., Bewegliche Stabwerke vom Bricardschen Typus, 1977.
Wunderlich, W., Gefährliche Annahmen der Trilateration und bewegliche Fachwerke I, 1977.
Wunderlich, W., Gefährliche Annahmen der Trilateration und bewegliche Fachwerke II, 1977.
Wunderlich, W., Über gefährliche Annahmen beim Clausenschen und Lambertschen Achtpunktproblem, 1978.
Wunderlich, W., Eine merkwürdige Familie von beweglichen Stabwerken, 1979. Wunderlich, W., Snapping and shaky antiprismas, 1979.
Wunderlich, W., Neue Wackelikosaeder, 1980.
Wunderlich, W., Wackelige Doppelpyramiden, 1980.
Wunderlich, W., Zur projektiven Invarianz von Wackelstrukturen, 1980.
Wunderlich, W., Wackeldodekaeder, 1980.
Wunderlich, W., Projective invariance of shaky structures. 1982.
Wunderlich, W., Kipp-Ikosaeder I, 1981.
Wunderlich, W., Kipp-Ikosaeder II, 1982.
Wunderlich, W., Ringartige Wackelpolyeder, 1982.
Wunderlich, W., Wackeldodekaeder. 1982.
Wunderlich, W., and Schwabe, Ch., Eine Familie von geschlossenen gleichflächigen Polyedern die fast beweglich sind, 1986.

Wunderlich, W., Fast bewegliche Oktaeder mit zwei Symmetrieebenen, 1987.
Wunderlich, W., Shaky polyhedra of higher connection, 1990.

A Review of Walter Wunderlich's Scientific Work

Walter Wunderlich's scientific work comprises 205 papers including three books. Almost all of the papers are journal papers and single authored. The main part of his scientific work is devoted to kinematics. More than 60 papers including one book are from this field. His other interests were in descriptive geometry, the theory of special curves and surfaces, the classical differential geometry and the application of geometry in surveying, civil and mechanical engineering.

It is worth noting that Walter Wunderlich did not develop an outstanding scientific theory, he contributed self-contained solutions to a large variety of problems in the above mentioned fields. All the time his unique geometric intuition was basic to his approach and sometimes the reader of his papers needs a lot of geometric knowledge to follow his elegant arguments and his stringent style of proofs. In the following review of Wunderlich's scientific achievements we will restrict ourselves to kinematic papers, the other fields will be mentioned only briefly in one subsection.

The first papers published by Wunderlich are devoted to kinematic problems in Non-Euclidean geometries. After F. Klein's Erlangen program from 1872, studies in different Non-Euclidean geometries became very popular. It was quite natural that also kinematic theory was developed in Non-Euclidean settings. The most important mathematicians and geometricians contributing to this field were W. Blaschke, E. Study, and F. von Lindemann. In W. Wunderlich's first scientific papers, essentially coming from his master's thesis, his dissertation and his habilitation thesis, he studies screw motions in elliptic space, hyperbolic space and the isotropic space which can be obtained from the elliptic space by some limit process.[3] Most of the time Wunderlich uses geometric or descriptive geometric methods to determine invariant properties of motions. To obtain graphic solutions he uses a Clifford parallel projection, which is quite natural in these Non-Euclidean Geometries. Most important for the classical kinematics in the plane are his results on screw surfaces in a

[3] This limiting case and the screw motion were also studied later on by Husty (1983) in his Dissertation.

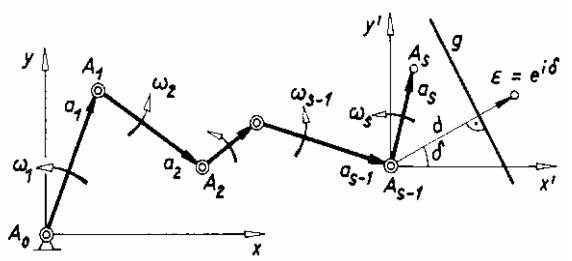

Fig. 2. Higher cycloidal motions.

quasi-elliptic space, because this Non-Euclidean space is the kinematic image space of planar motions.

In 1947 he published a paper dealing with a generalization of cycloidal motions (Wunderlich, 1947). In this paper he introduced so-called higher cycloidal motions. Cycloidal motions are obtained if both centrodes of a planar motion are circles. The paths of points are cycloids (epi-cycloids, hypo-cycloids). The same motion can be obtained by the end-effector of a planar 2R-linkage when both revolute joints rotate with constant angular velocity. Wunderlich's generalization is now that he allows n systems. Therefore he obtains the one-parameter motion of an nR-chain where all links rotate with constant angular velocity (see Figure 2).

The paths of points of the final system are called *cycloids of nth stage*. The analytic representation of the paths is obtained easily when one uses complex numbers to describe planar motions:

$$z = x + Iy = \sum_{i=1}^{s} a_v e^{I\omega_v t}.$$

The description of planar motions with complex numbers requires a historical remark: Although Bereis (1951) is often cited to be the founder of this method, Wunderlich has used complex numbers (also the so-called isotropic coordinates) earlier. In private communications he claimed that he was the one who encouraged Bereis to develop a theory of planar kinematics using complex numbers. According to Wunderlich it was the Italian geometrician Bellavitis (1874) who underlined the vectorial interpretation of complex numbers using the symbol "ramun" (*radice di meno uno*) instead of I to describe the 90° rotation. The application of isotropic coordinates was introduced by Cayley (1868) and Laguerre (1870). The main result of Wunderlich concern-

ing generalized cycloidal motions is the generalization of Euler's theorem: *Every cycloid of nth stage can be generated by a* $2(n-1)$*-chain of circles in* $n!$ *different ways*. Wunderlich shows that the moving and the fixed centrodes of the corresponding one-parameter higher cycloidal motions are higher cycloids and discusses a lot of geometric properties of the paths of points and the envelopes of lines. Wunderlich proves also another generalisation of Euler's theorem: *Every higher cycloid of nth stage can be generated in n different ways by rolling of two higher cycloids of stage* $n-1$. This theorem is visualized in Figure 3: a cycloidal motion is given by the equation:

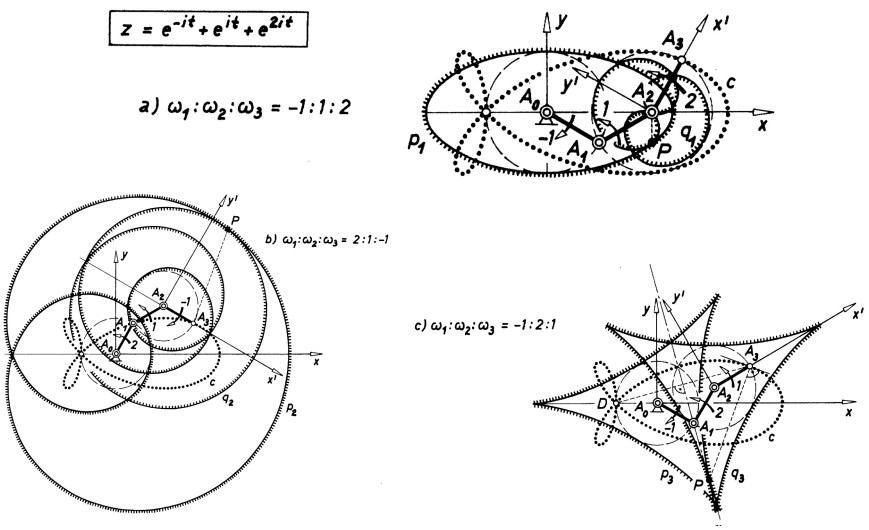

Fig. 3. Triple generation of a higher cycloidal curve of 3rd stage.

The path of A_3 in this motion is a rhodonea (rose-curve; the dotted curve in the three figures). The motion can be generated in three different ways in

(a) the fixed polhode is an ellipse and the moving polhode is Pascal's curve;
(b) an epi-cycloid rolls on a Pascal-curve; and
(c) the polhodes are two Steiner-cycloids.

Later on Wunderlich used higher cycloids for curve approximation in the plane (Wunderlich, 1950).

In 1968 Wunderlich published his book on planar kinematics (*Ebene Kinematik*). This book contains all topics of planar kinematics treated from a

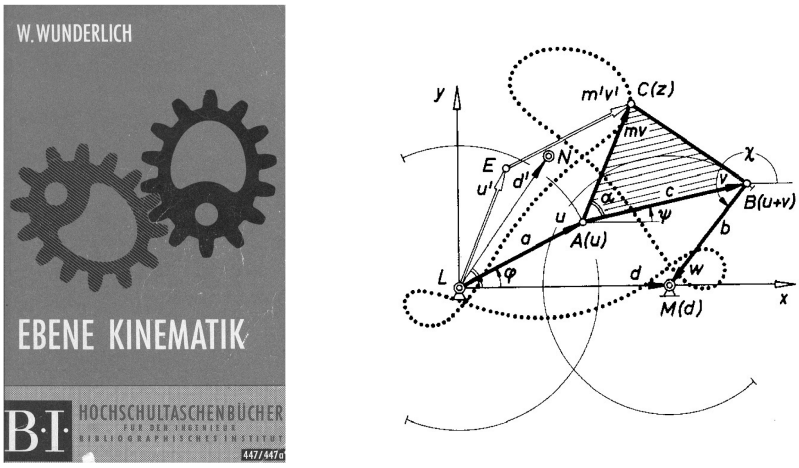

Fig. 4. Coupler curves.

geometric point of view, but it has many links to the applications and some unusual facets. It starts with the classical basics of planar kinematics but, a noteworthy point, complex numbers are used continuously to describe the motions and their properties analytically. The advantage of the method may be shown in Wunderlich's derivation of the equation of the coupler curve of a planar four-bar using isotropic coordinates.

Referring to the notation of Figure 4 he writes the bars LA, AB and BM of the four-bar in complex numbers. These complex numbers have constant absolute value (modulus) $|\mathbf{u}| = a$, $|\mathbf{v}| = c$, $|\mathbf{w}| = b$ but changing arguments (angles) and the constant sum

$$\mathbf{u} + \mathbf{v} + \mathbf{w} = d.$$

An arbitrary complex number $\mathbf{m}$ is then used to describe a point of the coupler system:

$$\mathbf{z} = \mathbf{u} + \mathbf{m}\mathbf{v}.$$

Using the relations:

$$\mathbf{u}\bar{\mathbf{u}} = \mathbf{a}^2, \quad \mathbf{v}\bar{\mathbf{v}} = \mathbf{c}^2, \quad \mathbf{w}\bar{\mathbf{w}} = \mathbf{b}^2$$

to eliminate $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ and their conjugates, he obtains a relation between $\mathbf{z}$ and $\bar{\mathbf{z}}$ which is the equation of the coupler curve in minimal (isotropic) coordinates:

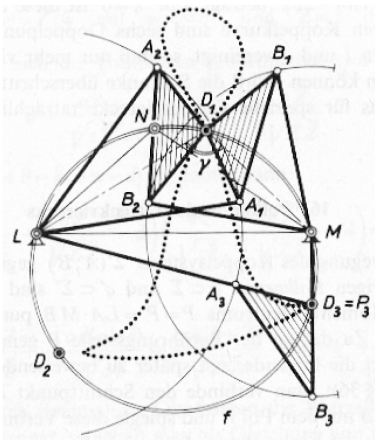

Fig. 5. Focal circle.

$$[\bar{\mathbf{n}}(\mathbf{z} - d)P - \bar{\mathbf{m}}\mathbf{z}Q][\mathbf{n}(\bar{\mathbf{z}} - d)P - \mathbf{m}\bar{\mathbf{z}}Q] + c^2R^2 = 0,$$

where P, Q, R are quadratic polynomials in $\mathbf{z}$, and $\bar{\mathbf{z}}$ and $\mathbf{n} = \mathbf{m} - 1$. This concept of minimal coordinates is used in the book consequently up to the third differential order to derive e.g. the equation of center-point curves and all the other well-known curves in planar kinematics.

Figures 5 and 6 show typical examples from the book on planar kinematics. There is no other book in planar kinematics which visualizes the theoretical concepts in such a clear way. Figure 5 deals with the focal circle. At first Wunderlich shows, using the complex representation of the coupler-curve, that the three points L, M and N forming a triangle similar to the coupler triangle are exceptional foci of the coupler curve, because they are intersections of isotropic asymptotes of the coupler curve. A simple symmetry argument in the two triangles A_1DA_2 and B_1DB_2 yields that D must be seen from L and M under the angle γ, therefore D must be on the focal circle f. Because f and the coupler curve, which is of degree six, have 12 points of intersection, from which six are in the circle-points, one can conclude immediately that each coupler curve has three double points. Figure 5 now shows clearly Wunderlich's ability to visualize the whole complexity of this theorem: in one figure, he displays all the possibilities that can happen. The double points can be real (D_1), complex (D_2) or the double point can be a cusp (D_3). Figure 6 presents an example of a symmetrical four-bar

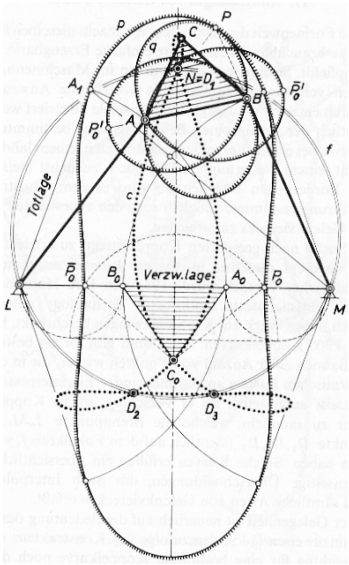

Fig. 6. Pole curves and coupler curves.

with coupler curve having three real double points and a moving and a fixed centrode. In a full chapter, applications and special layouts of four-bar mechanisms are discussed. Figure 7 shows how Wunderlich has linked in the book the geometric and kinematic theory with practical applications. On the left-hand side one can see the electro-mechanical device, namely a high voltage switch and on the right-hand side there is the kinematic analysis of the device containing the centrodes of the motion and the coupler curve of the point of interest P.

An important part of the book is devoted to multi-body mechanisms, focal mechanisms and singular planar multi-body systems. The investigation of singular frameworks is one of the main contributions of Walter Wunderlich. Most the time he provides simple, convincing geometric arguments for the exceptionality of a linkage. In Figures 8 and 9 two examples are shown. Both examples show nine-bar frameworks. In the first example two triangles ABC and LMN are linked by three bars AL, BM and CN and in the second we see a hexagon $ABCDEF$ with its three diagonals AD, BE and CF.

In both figures, from left to right, rigid, shaky (infinitesimal movable) and movable designs are shown. Without computations Wunderlich shows the shakiness in the first case by resorting to an old and well-known theorem

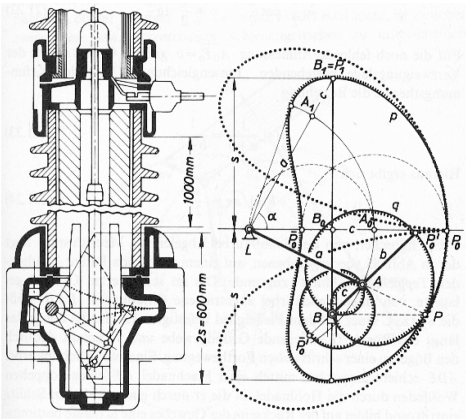

Fig. 7. High voltage switch.

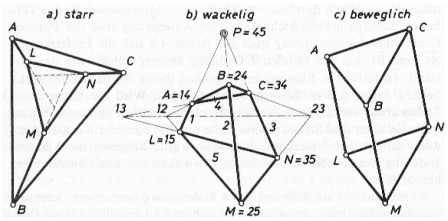

Fig. 8. Nine-bar framework, first layout.

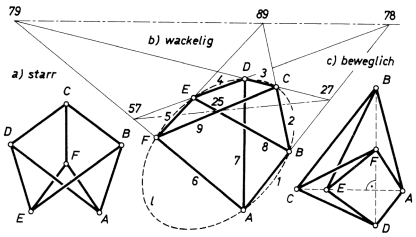

Fig. 9. Nine-bar framework.

in geometry, namely that the two triangles have to be in a Desarguian configuration and the movability yields the parallel bar mechanism. It should be noted that this mechanism became famous almost thirty years after Wunderlich published his book as the planar analogue of the Stewart–Gough platform, namely the 3-RPR planar parallel manipulator. The second framework

is in its movable realisation known as Dixon's mechanism. But the condition for shakiness is interesting. Wunderlich explains the condition again by resorting to the old theorem of Pappus–Pascal, which yields that the points A, B, C, D, E, F have to be on a conic section, when the framework is shaky. Wunderlich's early investigations on shakiness culminate in the first proof of the projective invariance of shaky structures. An earlier proof had been given by H. Liebmann, but with the restriction that the framework has to contain at least one triangle. Wunderlich shows that this assumption is not necessary and gives a relatively simple proof for the theorem (Wunderlich, 1980).

The book *Ebene Kinematik* also has a full chapter on the geometric theory of gears and cams. This is due to the fact that Wunderlich made significant contributions to this theory. Starting with the geometric theory of constructing gear profiles for constant transmission ratio, he develops the classical theory of involute gears due to F. Reuleaux, the theory of cycloidal gears due to Ch.E.L. Camus based on the earlier work of Ph. de la Hire. Remarkable is the paragraph on the geometry and kinematics of the Wankel motor.

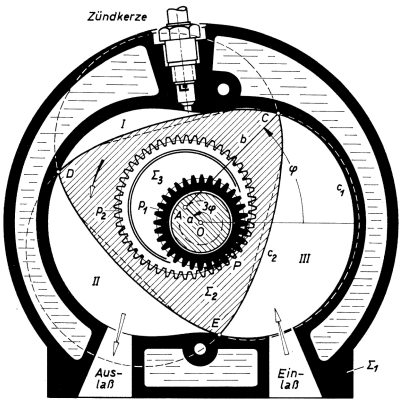

Fig. 10. Wankel motor.

Here one can see clearly Wunderlich's approach to difficult kinematic problems. One of the main problems of this motor type is the geometric form of the moving system (piston, Σ_2 in Figure 9). Wunderlich shows that this problem can be solved as a gear profile problem. Moreover he uses the so-called cyclographic mapping which maps circles into points of a three-

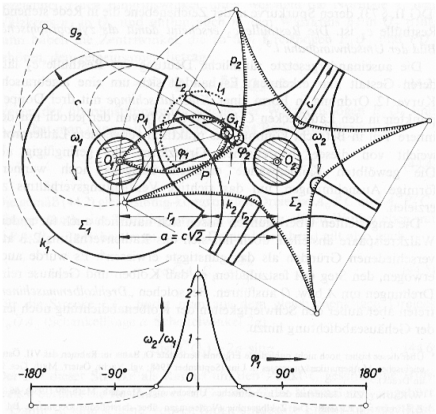

Fig. 11. Geneva wheel.

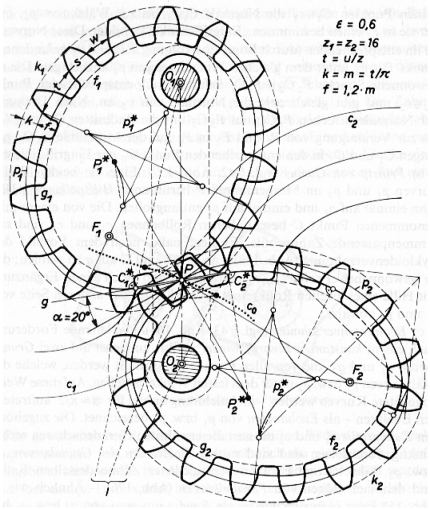

Fig. 12. Non-circular gears.

dimensional parameter space to investigate the geometric properties of the design curve of the piston.

In the book he discusses also in detail motion transmission with changing transmission ratio (non-circular gearing). As examples we take the Geneva wheel (Figure 11) and the involute gears with ellipses as base curves (Figure 12). Whereas the Geneva wheel is treated in many of the English text-

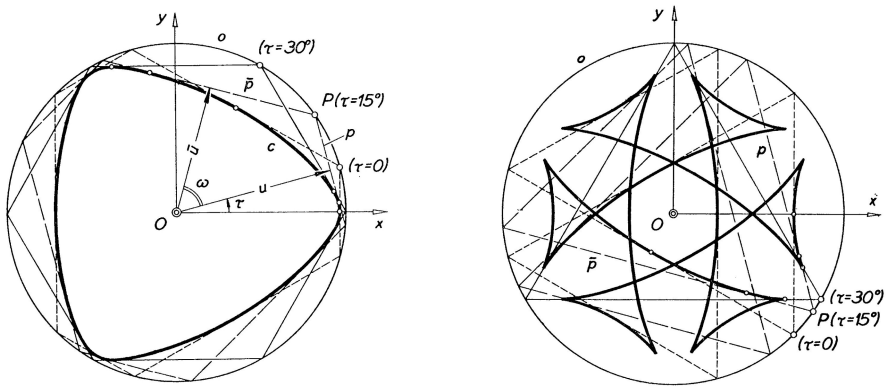

Fig. 13. Curves having an isoptic circle.

books on planar kinematics, the involute gear problem with non-circular base curves can be found (to the author's best knowledge) in none of the popular English textbooks. The elliptic gearing problem is also treated in Wunderlich and Zenow (1975).

Walter Wunderlich is best known to the community of mechanical engineers for his significant contributions to the theory of cams. Therefore it is not surprising that a full chapter of *Ebene Kinematik* is devoted to this problem. Although two papers (Wunderlich, 1971a, 1984) are published in English, the important papers dealing with the geometric basics of cams are published in German. In these papers he shows that the problem of designing single cams, that steer a flat-faced follower pair, is closely related to the construction of curves having an isoptic circle.[4] In Wunderlich (1971) he gives a complete solution to the problem which had been posed before (Green, 1950) but incompletely solved, and additionally he shows algebraic examples of such curves.

To finish the review of Wunderlich's book *Ebene Kinematik*, one has to mention the chapter on curvature theory of planar motions. He gives the first treatment of curvature theory in isotropic coordinates. But he does not limit to the mathematical description of all the well-known properties. The inflection circle (w in Figure 14), the circle point curve (k), the center-point curve (k^*) and Ball's point (U) are visualized for different types of mechanisms.

[4] An isoptic curve is the locus of points from which two tangents to the curve subtend a constant angle ω.

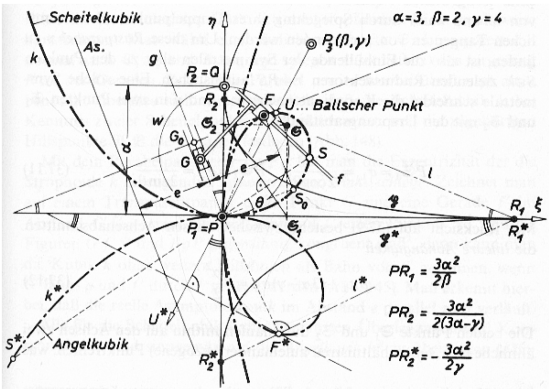

Fig. 14. Center-point curve, circle-point curve and Ball's point.

The most important kinematic topic of Wunderlich's later work are his contributions to overconstrained and shaky (infinitesimal movable) structures. He shows many remarkable examples but also develops methods that can be applied in today's research in singularity theory of nR-chains and parallel manipulators. Moreover he exhibits the close relation of shaky and pathologically movable structures to singularity problems in geodesy (see Wunderlich 1977, 1978). Already his first paper on "Rigid, snapping, shaky and movable octahedra", where he gives geometric conditions for all phenomena mentioned in the title, can be viewed as a complete singularity analysis of 3-3 Stewart–Gough manipulators (see Merlet, 2000). In the paper "Starre, kippende, wackelige und bewegliche Gelenkvierecke im Raum" one can find a complete singularity analysis of closed serial $4R$-chains up to the conditions for movability (Bennett-mechanism). This analysis is of course mathematically identical to the inverse kinematics problem of serial $4R$-manipulators.

In the papers "Fokalkurvenpaare in orthogonalen Ebenen und bewegliche Stabwerke" and "Bewegliche Stabwerke vom Bricardschen Typus" he corrects and improves an old result of Bricard concerning frameworks of rods connected by spherical joints: *Are* $m \geq 4, 5, 6$ *points* P_i *located on an ellipse and* $n \geq 6, 5, 4$ *points* Q_i *located on the corresponding focal hyperbola and all points* P_i *are connected to points* Q_i *by rigid rods, then this framework is movable even when all rods are connected by spherical joints*. Bricard claims that this framework has two degrees of freedom (dof) but Wunderlich proves that this is not correct: He shows that the correct dof is 3. Moreover he shows that one can take any pair of focal curves (focal curves of arbitrary degree,

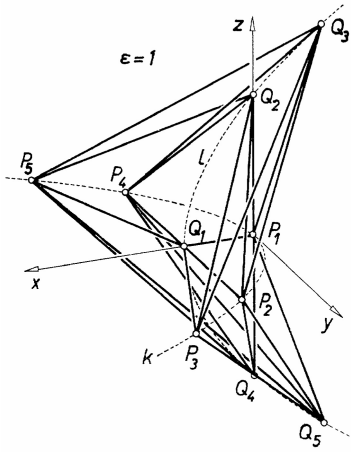

Fig. 15. Bricard framework.

not only ellipses and hyperbolas) and that the corresponding framework for arbitrary focal curves has 2 dofs.

In a whole series of papers he discusses the geometric conditions for infinitesimal movability or snapping[5] of polyhedra (icosahedra, dodecahedra, pyramids, prisms) and gives examples for each of the different cases. It must be mentioned that polyhedra having more then three edges in one face are considered as panel structures having revolute joints in the edges. Due to the basic theorem of Cauchy, all of these polyhedra have to be non-convex to allow infinitesimal movability. Therefore for each of such polyhedra there exists more then one assembly mode.

Figure 15 shows such a three-degrees-of-freedom framework. The knots are distributed on two focal parabolas.

Four of Wunderlich's papers on shakiness deserve special attention. In Wunderlich (1980d, 1982a) he proofs the projective invariance of shakiness of spatial frameworks. Projective invariance means: if a framework is shaky, then any linear transformation of its design will not resolve the shakiness. This includes of course any kind of projections. In his last paper (Wunderlich, 1990), written at the age of more than 80 years, and in Wunderlich (1982c) he gives examples of shaky polyhedra with genus > 0 (Figure 16). He provides

[5] This means that two assembly solutions are very close to each other and can be transformed into each other because of tolerances in the joints.

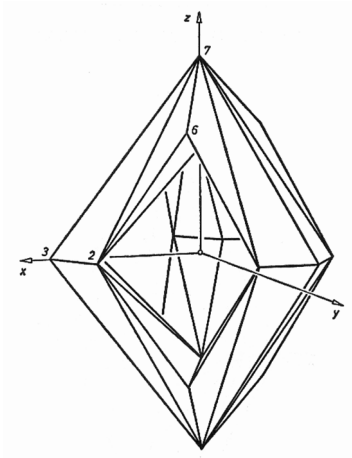

Fig. 16. Shaky structure of genus 3.

a geometric algorithm to generate shaky polyhedra of arbitrary genus. The existence of such structures had been doubted by all scientists working in the field.

Short Review of Wunderlich's Contribution to Other Fields

To show Wunderlich's versatility, a selection of other significant contributions will be presented. Through his whole scientific life he was investigating the geometry of curves and surfaces. This interest was awakened at the beginning of his scientific career when he worked on the descriptive geometry of non-Euclidean screw surfaces and spiral surfaces. The theory of spiral motion and the curves and surfaces generated by this motion is treated extensively in his book *Descriptive Geometry II*. The spiral motion is a generalization of a screw motion where instead of a rotation a planar spiral motion is concatenated with a translation along the spiral axis.

The generation of special surfaces with special motions is a traditional topic in Austrian geometry. Wunderlich especially contributed to this topic kinematic generations of J. Steiner's famous Roman surface. But there are also papers on kinematic generation of cubic ruled surfaces where a parabola is moved to describe the surface and a developable Möbius strip.

Another main area of Wunderlich's work is on surfaces with special fall lines (curves of steepest slope on a surface). Here he investigates surfaces

with planar fall lines, Roman surfaces with planar fall lines or surfaces with conic sections as fall lines.

Wunderlich's contributions to the geometry of special curves would fill another overview paper. But a short enumeration of some topics where Wunderlich has made excellent contributions is essential to understand the breadth of his interests: Pseudo-geodesic lines of cylinders and cones, D-curves on quadrics, principal tangent curves of special surfaces, loxodromic curves on different surfaces, irregular curves and functional equations, auto-involute curves, curves with constant global curvature, Zindler-curves, Bertrand curves, spherical curves and auto-polar curves.

Concerning Wunderlich's versatile interest in geometry one has to mention his papers "On the statics of the rope ladder", or "Geometric considerations on an apple skin" or "On the geometry of bird eggs".

Modern Interpretation of Main Contributions to Mechanism Design

Walter Wunderlich's work still exerts significant influence in kinematic research. Algebraic manipulation systems allow answering some of the questions he left open. As a recent example, we mention his work on Dixon's mechanisms. Dixon showed in 1899 that a linkage consisting of nine bars connected by revolute joints is paradoxically mobile when certain geometric conditions are fulfilled. There are two layouts that yield mobile linkages.

Wunderlich gives an elegant proof for the mobility, but for the number of assembly modes of the nine-bar linkage he could only conjecture that there should be eight. Only recently Walter and Husty (2007) proved that he was right. Moreover, in the same paper it was proven that the two linkage layouts already known to Dixon (Figures 17 and 18) are the only possible mobile layouts. Further generalisations of Wunderlich's papers are given by Stachel. In [Stachel 1997] he shows that Dixon's mechanisms can be transferred to spherical kinematics and, using the principle of transference, one obtains an over-constrained spatial mechanism. This mechanism is a two-degree-of-freedom paradoxical linkage.

Shakiness of polyhedral structures has become an important subject in the field of combinatorial geometry. Wunderlich's papers on rigidity have been reviewed and extended. The recently published second edition of the *Hand-*

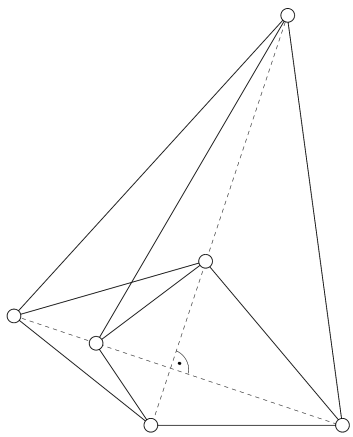

Fig. 17. Dixon Mechanism Type I.

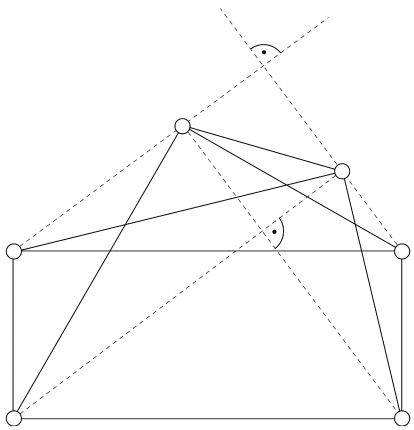

Fig. 18. Dixon Mechanism type II.

book of Discrete and Computational Geometry mentions 19 of Wunderlich's papers.

People working in the field of geodesy have applied Wunderlich's results in the theory of GPS navigation systems.

There is virtually no recent Austrian geometrician who has not used one of Wunderlich's results for his own scientific work. But of course Wunderlich himself was personally known to most of the now active scientists in kinematics and geometry, either as their teacher or reviewer of their theses

and papers. Unfortunately, because of language problems the dissemination of Wunderlich's results in the Anglophone world is rather small. It is hoped that this paper may help to advertise his results within this community.

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